Magneto-Hydrodynamic Flow of Fluid over a Stretching Porous Sheet

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CONTENTS

LIST OF SYMBOLS

Symbol	Denoted
B	Magnetic field strength vector
B0	Magnetic flux density along the y-axis,
J	Current density
m	Hall parameter
Pr	Prandtl number
M	Magnetic parameter
e	Unit electric charge,
E	Electric field
Re	Reynold's number
u,v,w	Components of velocity vector
C_p	Specific heat at constant temperature
T	Absolute free temperature of the fluid
T∞	Absolute temperature of the upper sheet
Tw	Absolute temperature of the stretching sheet,
Q	Heat source coefficient
ρ	Fluid density
β	Heat resource parameter
μ	Coefficient of viscosity
ν	Kinematic viscosity
λ	Permeability parameter
Θ	Dimensionless temperature
κ	Fluid thermal diffusion
k	Thermal conductivity of porous medium
S	Suction /injection parameter
St	Stratification parameter

LIST OF FIGURES

LIST OF TABLES

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ABSTRACT

In this thesis we study laminar boundary layer magneto hydrodynamic flow of an incompressible, viscous and electrically conducting fluid over a stretching sheet embedded in a porous medium taking into account the effects of Hall current in the presence of heat source. The equations of motion and heat transfer are transferred to non-dimensional system of ordinary differential equations that are solved numerically using mathematic software. The effects of various parameters on the velocity and temperature profiles are discussed and presented graphically using Harvard software.

Chapter One

Introduction

1.1 Overview

The study of the behavior of the boundary layer over stretching sheet occurs in many engineering applications and Manufacturing processes in the industry, such as, cooling of metallic plates and boundary layer flow over heat treated materials between feed roll and a windup roll, rolling and manufacturing of plastic films, and the aerodynamic extrusion of plastic sheets, manufacture of plastic and rubber sheets. Extension will bring in unidirectional orientation to a pop-up, and thus the quality of the final product largely depends on the flow and heat transfer mechanism. To this end, the analysis of momentum and heat transfer fluid within the liquid on a continuously stretching surface is important to acquire some basic understanding of these processes. Sakiadis [1-2] pioneered the study of boundary layer flow over a continuous solid surface moving with constant velocity. Because of the importance of the boundary layer flow over stretching surface, various aspects have been investigated by many authors and they have published several papers on the flow and heat transfer problems for stretching surfaces. Such as Dutta et al. [4] and Grubka and Bobba [5] studied the temperature field in the flow over a stretching surface subject to a uniform heat flux. Elbashbeshy [6] considered the case of a stretching surface with variable surface heat flux. Chen and Char [7] presented an exact solution of heat transfer for a stretching surface with variable heat flux. P. S. Gupta and A. S. Gupta [8] examined the heat and mass transfer for the boundary layer flow over a stretching sheet subject to suction and blowing.

Aboeldahab and Salem [9] investigated the effects of magnetic field, convective and radiative heat transfer on flow over a stretching surface with heat and mass transfer. Hinze [10] studied turbulent fluid motion in an irregular condition of flow; and showed that various quantities exhibited a random variation with time and space coordinates. Abo-eldahab and El-Gendy [11] studied the radiation effect on connective heat transfer in an electrically conducting fluid at a stretching surface with variable viscosity and uniform free steam. They showed that the flow characteristics are markedly affected by the variation of viscosity with temperature.

All these authors have studied the flow through non porous medium and neglected the effect of internal heat generation. The flow through Porous Media series is aimed at engineers and scientists who work and perform research in a wide variety of disciplines involving transport of matter and energy in porous media. This includes Civil Engineering, Hydrology, Mechanical Engineering, Chemical Engineering, Material Engineering, Food Industry, Petroleum Engineering, Agricultural Engineering, Biomedical Engineering, and Geothermal Engineering, to mention but a few. Fluid-filled porous media are ubiquitous in many natural and industrial systems. The working of these systems is controlled and/or affected by the movement of fluids, solutes, particles, electrical charges, and heat through them. Examples of natural porous media and corresponding processes are the flow of oil, gas, and water in oil reservoirs; the potential mobilization of methane in gas hydrates; the flow of Non-Aqueous Phase Liquids (NAPLs) in contaminated aquifers; the storage of CO2, nuclear waste, other hazardous wastes, and heat in the subsurface; the flow of fluids and solutes in biological tissues; and melting and metamorphism of snow. Examples of industrial porous media and corresponding processes are the drying of paper pulp, the absorbing of liquids in diapers and similar absorbing products, gas and water management in fuel cells, and the drying of foods, as well as water and solute movement in building materials, detergent tablets, textiles, foams, coatings, paper, and filters. Many physical, chemical, thermal, and biological processes (such as fluids flow, diffusion, capillarity, dissolution, absorption, clogging, degradation, shrinkage, swelling, fracturing, and flow of electrical charges) occur in these materials. For the design, operation, and maintenance of porous media systems, it is extremely important to understand these processes, describe them quantitatively (by mathematical models) and simulate them. Porous media processes are observed, studied, and modeled at a wide range of scales, from nano to micro scales, through the laboratory scale, to the field scale. Understanding above-mentioned transport phenomena, experimental studies of them, Theory and Applications of Transport in Porous Media and modeling them at different scales, as well as coping with the uncertainties that are inherent in such models. There is extensive literature on flow through porous media. There is extensive literature on flow through porous media that is governed by the generalized Darcy's law. For instance Yamamoto and Iwamura [12] investigated boundary layer flow of a Newtonian fluid through a highly porous medium. Later Raptis et a l[13] used these equations to study the influence of free convective flow and mass transfer on flow through a porous medium. Raptis and Perdikis [14] investigated oscillatory flow of a Newtonian fluid through a porous medium.

Vajravelu el [15] studied the heat transfer characteristics in laminar boundary layer of viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Abel et al [16] studied connective heat and mass transfer in viscoelastic fluid flow through a porous medium over a stretching sheet with variable viscosity. Bhargava el al.[17] has taken the problem of mixed convection micropolar fluid driven by a porous stretching sheet and found the solution by finite element method.

Rashad [18] has studied the radiative effects on heat transfer from a stretching surface in a porous medium .Kumar[19] considered radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. However , in an ionized fluid where the density is low thereby magnetic field intensity is very strong , the conductivity normal to the magnetic field is reduced due to spiraling of electrons and ions about the magnetic lines of force before collisions take place and current induced in direction normal to both the electric and magnetic fields. This phenomena is known as Hall effect. Watanabe and Pop [20] investigated the magneto hydrodynamic (MHD) boundary-layer flow over a continuously moving semi-infinite flat plate by taking into account the Hall currents. Aboeldahab [21] and Aboeldahab and Elbarbary [22] studied the Hall current effects on MHD free-convective flow past a vertical plate with mass transfer. Shit [23] investigated the Hall effects on MHD free convective flow and mass transfer over a stretching sheet in the presence of chemical reaction. Fakhar et al. [24] studied the Hall effects on the unsteady magneto hydrodynamic (MHD) flow of a third grade fluid without considering the heat and mass transfer phenomena. The effect of Hall currents on the steady MHD flow of Berger's fluid between two parallel electrically insulating infinite planes was carried out by Rana et al. [25]. Mandal [26]and Grosh [27] investigated effects of Hall current on MHD coquette flow between parallel plates in rotating system , Jana et al [28] analyzed the Hall effect in steady flow past an infinite porous flat plate .Abd El-Aziz [29] investigated the effect of Hall currents on the flow currents on the flow and heat transfer of an electrically conducting fluid over an unsteady stretching surface in the presence of a strong magnet .Aboeldahab and El Aziz [30] investigated the Hall current and Joule heating effects on electrically conducting fluid past a semi – infinite plate with strong magnetic field and heat generation /absorption. Jaber [31] studied the effect of Hall currents, radiation and variable viscosity on free connective flow past a semi – infinite continuously stretching plate , Jaber [32] studied the effect of Hall currents and variable fluid properties on MHD flow past a continuously stretching vertical plate in the presence of radiation. Jaber [33] studied the influence of Hall currents and viscous dissipation on MHD in a vertical porous channel rotates with a uniform angular velocity Ω about the axis normal to the plates. Now we purpose to study the effects of Hall currents and analyze the combined effects of various parameters on the velocity and the temperature.

1.2 Research Objectives:

1) Study the effect of internal heat generation or absorption.

2) Convert the partial differential equations of the governing equations into dimensionless nonlinear ordinary differential equations.

3) Study the effect of the magnetic field, Hall and heat source/sink parameters on the velocity and temperature fields.

1.3 Applications

The study of Magneto hydrodynamic (MHD) flow through porous media is of fundamental importance in a wide range of disciplines, including natural sciences and technology.

In Geophysics, the fluid core of the Earth and other planets is theorized to be a huge MHD dynamo that generates the Earth's magnetic field due to the motion of the molten rock. Such dynamos work by stretching magnetic field lines that thread through turbulent or sheared flows in a conductive fluid: the total length of magnetic field line in a particular volume determines the strength of the magnetic field, so stretching the field lines increases the magnetic field. MHD was originally applied to astrophysical and geophysical problems, where it is still very important, but more recently to the problem of fusion power, where the application is the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet, a problem that has not been solved to any degree of satisfaction. In engineering, MHD is related to engineering problems such as plasma confinement, liquid-metal cooling of nuclear reactors, and electromagnetic casting (among others).

In early 1990s, Mitsubishi built a boat, the 'Yamato', which uses a magneto hydrodynamic drive, is driven by a liquid helium-cooled superconductor, and can travel at 15 km/h.

MHD power generation fueled by potassium-seeded coal combustion gas showed potential for more efficient energy conversion (the absence of solid moving parts allows operation at higher temperatures), but failed due to cost prohibitive technical difficulties.

In civil and agricultural engineering, knowledge of flows through porous media is applied in the efficient layout of drainage systems for irrigation, and in the recovery of swampy areas. In geotechnical engineering and soil physics, studies of flows through porous media are used in predicting the water movement in clays and other surface-active soils. The chemical engineers and ceramic engineers may make use of the filtration and seepage properties of the porous materials used. The general equations governing MHD flow over a stretching surface embedded in a porous medium, which include the equation of conservation of mass, the equation of momentum, the equation of conservation of energy, are given in chapter 3.

Chapter Two

Preliminary Concepts

Fluid flow is an important part of many processes, including transporting materials from one point to another, mixing of materials, and chemical reactions. In this chapter we will introduce flow and explore concepts that effect on flow and use it in our equations.

2.1 What Is Flow?

The movement of liquids and gases is generally referred to as "flow," a concept that describes how fluids behave and how they interact with their surrounding environment for example, water moving through a channel or pipe, or over a surface. Flow can be either steady or unsteady, If all properties of a flow are independent of time, then the flow is steady; otherwise, it is unsteady. That is, steady flows do not change over time. An example of steady flow would be water flowing through a pipe at a constant rate. On the other hand, a flood or water pouring from an old-fashioned hand pump is example of unsteady flow.

Flow can also be either laminar (A mode of flow in which the fluid moves in layers along continuous, well-defined lines known as streamlines). Or turbulent (an irregular, disorderly mode of flow). Laminar flows are smoother, while turbulent flows are more chaotic. One important factor in determining the state of a fluid's flow is its viscosity, or thickness, where higher viscosity increases the tendency of the flow to be laminar.

Laminar flow is desirable in many situations, such as in drainage systems or airplane wings, because it is more efficient and less energy is lost. Turbulent flow can be useful for causing different fluids to mix together or for equalizing temperature.

However, such flows can be very difficult to predict in detail, and distinguishing between these two types of flow is largely intuitive.

2.1.1 Liquid Flow:

The study of liquid flow is called hydrodynamics. While liquids include all sorts of substances, such as oil and chemical solutions, by far the most common liquid is water and most applications for hydrodynamics involve managing the flow of this liquid. That includes flood control, operation of city water and sewer systems, and management of navigable waterways.

Hydrodynamics deal primarily with the flow of water in pipes or open channels. Geology professor John Southard's lecture notes from an online course, "Introduction to Fluid Motions" (Massachusetts Institute of Technology, 2006), outline the main difference between pipe flow and open-channel flow: "flows in closed conduits or channels, like pipes or air ducts, are entirely in contact with rigid boundaries," while "open-channel flows, on the other hand, are those whose boundaries are not entirely a solid and rigid material." He states, "Important open-channel flows are rivers, tidal currents, irrigation canals, or sheets of water running across the ground surface after a rain."

Due to the differences in those boundaries, different forces affect the two types of flows. According to Scott Post in his book, "Applied and Computational Fluid Mechanics," (Jones & Bartlett, 2009), "While flows in a closed pipe may be driven either by pressure or gravity, flows in open channels are driven by gravity alone." The pressure is determined primarily by the height of the fluid above the point of measurement. For instance, most city water systems use water towers to maintain constant pressure in the system. This difference in elevation is called the hydrodynamic head. Liquid in a pipe can also be made to flow faster or with greater pressure using mechanical pumps.

2.1.2 Gas Flow:

The flow of gas has many similarities to the flow of liquid, but it also has some important differences. First, gas is compressible, whereas liquids are generally considered to be incompressible. In "Fundamentals of Compressible Fluid Dynamics" (Prentice-Hall, 2006), author P. Balachandran describes compressible fluid, stating, "If the density of the fluid changes appreciably throughout the flow field, the flow may be treated as a compressible flow." Otherwise, the fluid is considered to be incompressible. Second, gas flow is hardly affected by gravity.

The gas most commonly encountered in everyday life is air; therefore, scientists have paid much attention to its flow conditions. Wind causes air to move around buildings and other structures, and it can also be made to move by pumps and fans.

One area of particular interest is the movement of objects through the atmosphere. This branch of fluid dynamics is called aerodynamics, which is "the dynamics of bodies moving relative to gases, especially the interaction of moving objects with the atmosphere," according to the American Heritage Dictionary. Problems in this field involve reducing drag on automobile bodies, designing more efficient aircraft and wind turbines, and studying how birds and insects fly.

2.2 Viscosity:

2.2.1 Viscosity Definition:

The viscosity of a fluid is a measure of its resistance to gradual deformation by shear stress or tensile stress; it corresponds to the informed concept “thickness”. A fluid with a low viscosity is said to be "thin," while a high viscosity fluid is said to be "thick." It is easier to move through a low viscosity fluid than a high viscosity fluid. As a simple example; syrup has a much higher viscosity than water: more force is required to move a spoon through a jar of syrup than in a jar of water because the syrup is more resistant to flowing around the spoon. This resistance is due to the friction produced by the fluid’s molecules and affects both the extent to which a fluid will oppose the movement of an object through it and the pressure required to make a fluid move through a tube or pipe. Viscosity is affected by a number of factors, including the size and shape of the molecules, the interactions between them, and temperature. Most common fluids, called Newtonian fluids, have a constant viscosity. There is a greater resistance as you increase the force, but it's a constant proportional increase. In short, a Newtonian fluid keeps acting like a fluid, no matter how much force is put into it.

In contrast, the viscosity of non-Newtonian fluids is not constant, but rather varies greatly depending on the force applied. A classic example of a non-Newtonian is Oobleck(is one of the easiest types of slime you can make. It normally behaves like a liquid or jelly, but if you squeeze it in your hand, it will seem like a solid.), which exhibits solid-like behavior when a large amount of force is used on it. Another type of non-Newtonian fluid are known as magnetorheological fluids, which respond to magnetic fields by becoming nearly solid but reverting to their fluid state when removed from the magnetic field.

2.2.2 Kinematic Viscosity:

The kinematic viscosity is another way to look at the viscosity which means: property of liquids and gases that represents how easily a given substance can flow. In practical terms, it is closely related to how thick the substance is. Both absolute and kinematic viscosity change according to temperature. The reason for this new definition is that some experimental data are given in this form. To obtain kinematic viscosity ν, the absolute viscosity μ of a substance is divided by its density ρ:

$$\nu = \frac{\mu}{\rho} \tag{2.1}$$

Absolute viscosity, also called dynamic viscosity, measures a substance's resistance to flow. It is determined experimentally by sandwiching a liquid or gas between two plates and applying a known amount of pressure to move the top plate .The dynamic viscosity depends on the pressure, the amount of time it was applied, and the distance the plate moved in that time. Dynamic or absolute viscosity is based on the International System of Units (SI) units of Pascal-seconds (Pa*s), which means that if a pressure of 1 Pa is applied for 1 second, the plate will move the same distance as the distance between the two plates. Centipoise (cP) is also a common unit for dynamic viscosity , 1 cP is the viscosity of water around room temperature.

Density measures the mass of a substance relative to its volume, which means that it has units of mass per volume. The units are kg/m^3 in SI units or slugs/ft^3 in imperial units. Density can be understood by comparing it to weight — a piece of a denser material will weigh more than the same-size piece of a less dense material.

Since kinematic viscosity is dynamic viscosity divided by density, it has units of square meters per second (m^2/s) in the SI system or square feet per second (ft^2/s) in the imperial system. As for absolute viscosity, the imperial units are almost never used.

Heat affects material properties, so both types of viscosity change at higher temperatures. When a liquid is heated, it flows more easily and thus the viscosity decreases. Kinematic viscosity is somewhat less affected than absolute viscosity, as heat also reduces the density because molecules move farther apart as a substance is heated. The viscosity of gases increases at higher temperatures as a gas expands; it exerts more pressure on the plate, making it harder to move.

2.3: Magnetic Field:

2.3.1: Magnetic Field Basics:

Magnetic fields are different from electric fields. Although both types of fields are interconnected, they do different things. The idea of magnetic field lines and magnetic fields was first examined by Michael Faraday and later by James Clerk Maxwell. Both of these English scientists made great discoveries in the field of electromagnetism.

Magnetic fields are areas where an object exhibits a magnetic influence. The fields affect neighboring objects along things called magnetic field lines. A magnetic object can attract or push away another magnetic object. We also need to remember that magnetic forces are not related to gravity. The amount of gravity is based on an object's mass, while magnetic strength is based on the material that the object is made of. If you place an object in a magnetic field, it will be affected, and the effect will happen along field lines. Many classroom experiments watch small pieces of iron (Fe) line up around magnets along the field lines. Magnetic poles are the points where the magnetic field lines begin and end. Field lines converge or come together at the poles. You have probably heard of the poles of the Earth. Those poles are places where our planets field lines come together. We call those poles north and south because that's where they're located on Earth. All magnetic objects have field lines and poles. It can be as small as an atom or as large as a star.

2.3.2 Attracted and Repulsed

We know about charged particles. There are positive and negative charges. You also know that positive charges are attracted to negative charges. A French scientist named Andre-Marie Ampere studied the relationship between electricity and magnetism. He discovered that magnetic fields are produced by moving charges (current). And moving charges are affected by magnets. Stationary charges, on the other hand, do not produce magnetic fields, and are not affected by magnets. Two wires, with current flowing, when placed next to each other, may attract or repel like two magnets. It all has to do with moving charges.

Often the magnetic field is defined by the force it exerts on a moving charged particle. It is known from experiments in electrostatics that a particle of charge q in an electric field experiences a force F = qE. However, in other situations, such as when a charged particle moves in the vicinity of a current-carrying wire, the force also depends on the velocity of that particle. Fortunately, the velocity dependent portion can be separated out such that the force on the particle satisfies the Lorentz force law,

$$F = q[E + (v \times B)] \tag{2.2}$$

Here v is the particle's velocity and × denotes the cross product. The vector B is termed the magnetic field, and it is defined as the vector field necessary to make the Lorentz force law correctly describe the motion of a charged particle. The unit of B is (newton·second)/(coulomb·metre)

2.4 Hall Current:

The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. It was discovered by Edwin Hall in 1879.

The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field. It is a characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carriers that constitute the current.

2.4.1 Theory:

The Hall Effect is due to the nature of the current in a conductor. Current consists of the movement of many small charge carriers, typically electrons, holes, ions or all of the three. When a magnetic field is present, these charges experience a force, called the Lorentz force. When such a magnetic field is absent, the charges follow approximately straight, 'line of sight' paths between collisions with impurities, phonons, etc. However, when a magnetic field with a perpendicular component is applied, their paths between collisions are curved so that moving charges accumulate on one face of the material. This leaves equal and opposite charges exposed on the other face, where there is a scarcity of mobile charges. The result is an asymmetric distribution of charge density across the Hall element, arising from a force that is perpendicular to both the 'line of sight' path and the applied magnetic field. The separation of charge establishes an electric field that opposes the migration of further charge, so a steady electrical potential is established for as long as the charge is flowing.

In classical electromagnetism electrons move in the opposite direction of the current I (by convention "current" describes a theoretical "hole flow"). In some semiconductors it appears "holes" are actually flowing because the direction of the voltage is opposite to the derivation below.

For a simple metal where there is only one type of charge carrier(electrons) the Hall voltage V_H can be derived by using the Lorentz force and seeing that in the steady-state condition charges are not moving in the y-axis direction because the magnetic force on each electron in the y-axis direction is cancelled by an y-axis electrical force due to the buildup of charges. The v_x term is the drift velocity of the current which is assumed at this point to be holes by convention. The $v_x B_z$ term is negative in the y-axis direction by the right hand rule.

$F = q\,[E + (v \times B)]$.

$0 = E_y - v_x B_z$ where E_y is assigned in direction of y-axis .

In wires, electrons instead of holes are flowing, so $v_x = -v_x$ and $q = -q$

$$E_y = \frac{-v_H}{w} \tag{2.3}$$

Substituting these changes gives

$$V_{H=}\ \mathrm{v}_x B_z \mathrm{w} \tag{2.4}$$

The conventional "hole" current is in the negative direction of the electron current and the negative of the electrical charge which gives $I_x = ntw\,(-\mathrm{v}_x)(-e)$ where n is charge carrier density, t_w is the cross-sectional area, and -e is the charge of each electron. Solving for w and plugging into the above gives the Hall voltage:

$$V_H = \frac{\mathrm{I}_{xB_Z}}{n\mathrm{te}} \tag{2.5}$$

If the charge build up had been positive (as it appears in some semiconductors), then the V_H assigned in the image would have been negative (positive charge would have built up on the left side).

The Hall coefficient is defined as

$$\mathrm{R}_H = \frac{E_y}{j_{xB_z}} \tag{2.6}$$

Where j is the current density of the carrier electrons, and E_y is the induced electric field. In SI units, this becomes

$$\mathrm{R_H} = \frac{E_y}{j_{xB_z}} = \frac{tV_H}{IB} = \frac{1}{-n_e} \tag{2.7}$$

(The units of RH are usually expressed as m3/C, or Ω·cm/G, or other variants.) As a result, the Hall effect is very useful as a means to measure either the carrier density or the magnetic field.[34]

One of the most important features of the Hall Effect is that it differentiates between positive charges moving in one direction and negative charges moving in the opposite. The Hall effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons. The Hall effect also showed that in some substances (especially p-type semiconductors), it is more appropriate to think of the current as positive "holes" moving rather than negative electrons. A common source of confusion with the Hall Effect is that holes moving to the left are really electrons moving to the right, so one expects the same sign of the Hall coefficient for both electrons and holes. This confusion, however, can only be resolved by modern quantum mechanical theory of transport in solids. [35]

The sample inhomogeneity might result in spurious sign of the Hall Effect, even in ideal van der Pauw configuration of electrodes. For example, positive Hall effect was observed in evidently n-type semiconductors. Another source of artifact, in uniform materials, occurs when the sample's aspect ratio is not long enough: the full Hall voltage only develops far away from the current-introducing contacts, since at the contacts the transverse voltage is shorted out to zero.

2.5 Magneto Hydrodynamics (MHD):

(Magneto fluid dynamics or hydro magnetics) is the study of the magnetic properties of electrically conducting fluids. Examples of such magneto-fluids include plasmas, liquid metals, and salt water or electrolytes. The word magneto hydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning water, and dynamics meaning movement. The field of MHD was initiated by Hannes Alfvén, for which he received the Nobel Prize in Physics in 1970.

The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations must be solved simultaneously, either analytically or numerically.

2.6 Heat Flux:

Heat Flux is the rate of heat energy that passes through a surface. The SI derived unit of heat rate is joule per second, or watt. Heat flux density is the heat rate per unit area. In SI units, heat flux density is measured in [W/m2]. Heat rate is a scalar quantity, while heat flux is a vectorial quantity. To define the heat flux at a certain point in space, one takes the limiting case where the size of the surface becomes small.

Heat fluxes are everywhere. Some examples are:

- Getting cold feet from standing on a cold floor: since the floor has a lower temperature than the feet, heat flows from the feet to the floor.
- Standing close to a fire feels hot: the temperature of a fire is much higher than the surrounding air. Therefore, heat radiates from the fire to the surroundings.
- Feeling hot in a sauna: since the air temperature in a sauna is higher than the body’s temperature, heat flows from the air into the body.

2.6.1 Types of Heat Transport:

In order for heat flux to exist, it requires, not only a temperature difference, but also a medium through which heat is flowing. Heat can flow through solid materials (in which case it is called conduction), through gases and liquids (which is called convection) and through electromagnetic waves (which is called radiation).

Examples of conductive heat flux are (through solid materials):

- Touching a hot cup of coffee
- Thermal influences in precision instruments.
- Measurement of heat output from chemical reactors.

Examples of convective heat flux are (through liquids and gases):

- Feeling much colder when it is windy.
- Feeling much colder in water of 25°C than in air of 25°C.
- Sensing principle in heat flux based mass flow sensors.

Examples of radiation heat flux are (electromagnetic waves):

- Cooking food in solar oven.
- Feeling the heat from camp fire.

2.6.2: Relationship Between Heat Flow & T Gradient: Fourier's Law:

The rate of heat flow is proportional to the difference in heat between two bodies. A thin plate of thickness z with temperature difference ΔT experiences heat flow Q [36]:

$$Q = -K\frac{\Delta T}{z} \tag{2.8}$$

where k is a proportionality constant called the thermal conductivity (J/msK):

We can express the above equation as a differential by assuming that z→0:

$$Q(z) = -K\frac{\partial T}{\partial z} \qquad (2.9)$$

(We use a minus sign because heat flows from hot to cold and yet we want positive T to correspond to positive x, y, z.)

In other words, the heat flow at a point is proportional to the local slope of the T–z curve

If the temperature is constant with depth ($\partial T/\partial z = 0$), there is no heat flow—of course! Moreover, if $\partial T/\partial z$ is constant (and nonzero) with depth ($T(z)=T_{z0} + mz$), the heat flow generalized to 3D, the relationship between heat flow and temperature is:

$$Q = -k\Delta T = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right) \qquad (3.10)$$

i.e., the heat flow at a point is proportional to the local temperature gradient in 3D.
Of course, if the heat flow is not constant with depth, the temperature must be changing. The temperature at any point changes at a rate proportional to the local gradient in the heat flow:

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho C_p}\frac{\partial Q}{\partial z} \qquad (2.11)$$

So, if there is no gradient in the heat flow ($\frac{\partial Q}{\partial z} = 0$), the temperature does not change. If we then stuff the equation defining heat flow as proportional to the temperature gradient

($\mathrm{Q} = -\mathrm{k}\frac{\partial T}{\partial z}$) into the equation expressing the rate of temperature change as a function of the heat flow gradient ($\frac{\partial T}{\partial z}$ and $\frac{\partial Q}{\partial z}$), we get the rate of temperature change as a function of the curvature of the temperature gradient (perhaps more intuitive than the previous equation):

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial z^2} \quad (2.12)$$

And, in 3D, using differential operator notation (∇^2 is known as 'the Laplacian'):

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\nabla^2 T \quad (2.13)$$

This is the famous 'diffusion equation, it can be expressed most efficiently as:

$$\frac{\partial T}{\partial t} = \kappa\, \nabla^2 T \quad (2.14)$$

where κ is the thermal diffusivity:

$$\kappa = \frac{k}{\rho C_p} \quad (2.15)$$

2.7 Remark:

Heat transfer occurs at a lower rate across materials of low thermal conductivity than across materials of high thermal conductivity. Correspondingly, materials of high thermal conductivity are widely used in heat sink applications and materials of low thermal conductivity are used as thermal insulation. The thermal conductivity of a material may depend on temperature. The reciprocal of thermal conductivity is called thermal resistivity.

Thermal conductivity is actually a tensor, which means it is possible to have different values in different directions.

There are a number of ways to measure thermal conductivity. Each of these is suitable for a limited range of materials, depending on the thermal properties and the medium temperature. There is a distinction between steady-state and transient techniques.

In general, steady-state techniques are useful when the temperature of the material does not change with time. This makes the signal analysis straightforward (steady state implies constant signals). The disadvantage is that a well-engineered experimental setup is usually needed. The Divided Bar (various types) is the most common device used for consolidated rock solids.

2.8 The Boussinesq Approximation:

The Boussinesq approximation is applied to problems where the fluid varies in temperature from one place to another, driving a flow of fluid and heat transfer. The fluid satisfies conservation of mass, conservation of momentum and conservation. In the

Boussinesq approximation, variations in fluid properties other than density ρ are ignored, and density only appears when it is multiplied by g, the gravitational acceleration [37].

2.8.1The Continuity Equation:

A continuity equation in physics is an equation that describes the transport of some quantity. It is particularly simple and particularly powerful when applied to a conserved quantity, but it can be generalized to apply to any extensive quantity. Since mass, energy, momentum, electric charge and other natural quantities are conserved under their respective appropriate conditions; a variety of physical phenomena may be described using continuity equations.

The equation of continuity is derived from the law of conservation of mass. The law of conservation of mass assumes that mass can neither be created nor destroyed and that on a steady flow process, the stored mass in a control volume does not change. A steady flow process is one where the flow rate does not change over time. This implies that inflow into the control volume equals outflow. For a steady fluid flow, the form of the equation of continuity is:

$$\frac{\partial \rho}{\partial t}+\frac{\partial (\rho \mathrm{u})}{\partial x}+\frac{\partial (\rho \mathrm{v})}{\partial y}+\frac{\partial (\rho \mathrm{w})}{\partial z}=0 \tag{2.16}$$

Since the fluid in consideration is assumed to be incompressible (i.e constant density), then equation of continuity takes the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}+\frac{\partial \mathrm{w}}{\partial z}=0 \tag{2.17}$$

2.8.2 Momentum Equation :

The law of the conservation of momentum states that the rate of change of momentum in the control volume is equal to the sum of the net momentum flux into the control volume and any external forces acting on the control volume. This implies that the total momentum of a closed system of objects is constant.

Although the most rigorous derivation of the conservation of momentum equations also stems from the general form continuity equation formed above, a quicker and nearly as rigorous derivation can be done using Newton's laws and an application of the chain rule. Basic physics dictates that

$$\vec{F} = m\,\vec{a} \tag{2.18}$$

Allowing for the body force $\vec{F} = \vec{b}$ and substituting density for mass, we get a similar equation:

$$\vec{b} = \rho \frac{d}{dt}\,\vec{\mathrm{v}}\,(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) \tag{2.19}$$

Note that we can substitute density for mass because we are operating with a fixed control volume and infinitesimal fluid parcels.

The body force $\vec{\mathrm{b}}$ is a force that acts throughout the body of fluid (as opposed to, say, a shear force, which acts parallel to a plane).

Applying the chain rule to the derivative of velocity, we get

$$\vec{b} = \rho\left(\frac{\overrightarrow{\partial \mathrm{v}}}{\partial t} + \frac{\overrightarrow{\partial \mathrm{v}}}{\partial x}\frac{\partial x}{\partial t} + \frac{\overrightarrow{\partial \mathrm{v}}}{\partial y}\frac{\partial y}{\partial t} + \frac{\overrightarrow{\partial \mathrm{v}}}{\partial z}\frac{\partial z}{\partial t}\right) \tag{2.20}$$

Equivalently,

$$\vec{b} = \rho\left(\frac{\overrightarrow{\partial v}}{\partial t} + \vec{\mathrm{v}}.\nabla\vec{\mathrm{v}}\right) \tag{2.21}$$

Substituting the value in parentheses for the definition of a material derivative, we obtain our final equation of

$$\rho \frac{D\vec{V}}{Dt} = \vec{b} \qquad (2.22)$$

2.8.3 Equation of Conservation of Energy:

For a single phase material, the amount of heat per unit volume is $\Phi = \rho c_p T$ where c_p is the specific heat (energy per unit mass per degree Kelvin) at constant pressure and T is the temperature. The heat flux has two components due to conduction and transport. In the absence of transport the heat flux is $F = -k\nabla T$ where k is the thermal conductivity. Note that heat flows opposite to ∇T, i.e. heat flows from hot to cold. The transport flux is $c_p TV$. Finally, unlike mass, heat can be created in a region due to terms like radioactive decay or viscous dissipation and shear heating. We will just lump all the source terms into H. Thus the simplest conservation of heat equation is

$$\frac{\partial \rho c_p T}{\partial t} + \nabla.(\rho c_p TV) = k\nabla.\nabla T + H \qquad (2.23)$$

For constant c_p and k, this equation can also be rewritten:

$$\frac{\partial T}{\partial t} + V\nabla T = \kappa\nabla^2 T + H/\rho c_p \qquad (2.24)$$

Where $\kappa = k/\rho c_p$ is the thermal diffusivity.

The three numbered equations are the basic convection equations in the Boussinesq approximation.

Chapter Three

MHD Flow over a Stretching Surface in Porous Medium with Heat Transfer

3.1 Governing Equations and Analysis:

In this chapter we shall discuss the equations governing the MHD flow of an electrically conducting fluid over a horizontal sheet in porous medium. The equations of the conservation of momentum and the equation of energy are derived. This is followed by non-dimensionalizing process of the equations governing the flow. Newton' method used to approximate the solution to the governing equations is then discussed. Finally the results are discussed at the end of the chapter.

Consider the incompressible viscous laminar flow caused by moving sheet, which is placed in stationary fluid, in presence of heat source. The flow is assumed to be in the x-direction which is chosen along the sheet and the y-axis perpendicular to it. A transverse magnetic field of strength B_0 is applied parallel to y-axis. The magnetic Reynolds number is taken to be small enough so the induced magnetic field is negligible in comparison with the applied magnetic field, so that:

$B_x = B_z = 0$ and $B_y = B_0$

The viscous and joule heating are taken in account. The generalized Ohm's law [38] including Hall current is given in the form:

$$J = \frac{\sigma}{1+m^2}\left(E + V \times B - \frac{1}{en_e} J \times B \right) \tag{3.1}$$

where J is the electric current density (J_x , J_y , J_z) are components of electric current density J, the equation of conservation of electric charge $\nabla . J = 0$ yields $J_y = 0$, V is the velocity vector, E is the intensity vector of the electric field, B is the induced magnetic vector , m is the Hall parameter and e is the charge of an electron, n_e is the number density of electrons. Neglecting polarization effect, the electric field E is given as E = 0.

So

$$J = (J_x, 0, J_z) , B = (0, B_0, 0), V = (u, v, w) \tag{3.2}$$

$$J_x = \frac{\sigma B_\circ}{(1+m^2)} (mu + v) \tag{3.3}$$

$$J_y = \frac{\sigma B_\circ}{(1+m^2)} (mv - u) \tag{3.4}$$

From equations above and considering the usual boundary layer and under Boussinesq approximations, the equations of momentum, concentration, and energy in porous medium are given below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \frac{\sigma B_0^2}{\rho(1+m^2)} (u + mw) \tag{3.6}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)} (mu - w) \tag{3.7}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T_\infty - T) \tag{3.8}$$

The boundary conditions are:

$$At\ \ y = 0 \quad u = cx\ , \mathrm{v} = -\mathrm{v}_0\ ,\ w = 0 \tag{3.9}$$

$$At\ y = \infty \quad u = \mathrm{v} = w = 0 \tag{3.10}$$

In addition to the above , boundary conditions on the temperature are:

$$\frac{\partial T}{\partial y} = Bx^2 \text{ at y} = 0\ , T = T_\infty \text{ as } y \longrightarrow \infty \tag{3.11}$$

Introducing the following non-dimensional variables:

$$\mathrm{u} = cxf'(\eta)\ ,\ \ \mathrm{V} = -\sqrt{\nu c}\, f(\eta)\ \ ,\ \ \eta = \sqrt{\frac{c}{\nu}}\, y\ ,\ w = cxh(\eta) \tag{3.12}$$

We get:

$$\frac{\partial u}{\partial x} = cf' \tag{3.13}$$

$$\frac{\partial u}{\partial y} = cx\sqrt{\frac{c}{\nu}}\, f'' \tag{3.14}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{c^2 x}{\nu} \quad f''' \tag{3.15}$$

$$\frac{\partial w}{\partial x} = ch \tag{3.16}$$

$$\frac{\partial w}{\partial y} = cx\sqrt{\frac{c}{\nu}}\ h' \tag{3.17}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{c^2}{\nu} xh'' \tag{3.18}$$

Substitute equations 3.13 – 3.15 in equation (3.6) ,we get:

$$c^2xf'f' - \sqrt{\nu c}\, f\, cxf''\sqrt{\frac{c}{\nu}} = \nu\,\frac{c^2x}{\nu}\; f''' - \frac{\nu}{K}\, cx\, f' - \frac{\sigma\beta_0^2}{\rho(1+m^2)}(cxf' + mcxh)$$

Thus,

$$c^2x\;\, f'^2 - c^2xff'' = c^2x\;\, f''' - \frac{\nu cx}{K} f' - \frac{\sigma\beta_0^2 cx}{\rho(1+m^2)} f' - \frac{\sigma\beta_0^2 cx\, m}{\rho(1+m^2)} h \quad (3.19)$$

Divide by c^2x we get:

$$f'^2 - ff'' = f''' - \frac{\nu}{cK} f' - \frac{\sigma\beta_0^2}{c\rho(1+m^2)} f' - \frac{\sigma\beta_0^2}{\rho c}\;\frac{m}{(1+m^2)} h \qquad (3.20)$$

Where prime denotes differentiation with respect to η only and the dimensionless parameters are $M = \frac{\sigma\beta_0^2}{\rho c}$ the magnetic parameter and $\lambda = \frac{\nu}{cK}$ is defined as the permeability parameter.

Equation (3.6) becomes:

$$f'^2 - ff'' = f''' - \lambda\, f' - \frac{M}{(1+m^2)}\,(f' + mh) \qquad (3.21)$$

Substitute equations 3.16 – 3.18 in equation (3.7), we get:

$$cx\, f'ch - \sqrt{\nu c}\, fcxh'\sqrt{\frac{c}{\nu}} = \frac{c^2x}{\nu}\; h'' + \frac{\sigma\beta_0^2}{\rho(1+m^2)}(mcxf' - cxh) \qquad (3.22)$$

Divide by c^2x, we get:

$$f'h - fh' = h'' + \frac{\sigma\beta_0^2}{\rho c(1+m^2)}(mf' - h) \tag{3.23}$$

Equation (3.7) becomes:

$$f'h - fh' = h'' + \frac{M}{(1+m^2)}(mf' - h) \tag{3.24}$$

To solve the equation (3.8) with corresponding to boundary condition (3.11), we assume the dimensionless temperature $\theta(\eta)$as :

$$T = T_\infty + B\sqrt{\frac{\nu}{c}}\, x^2\theta(\eta) \tag{3.25}$$

$$\frac{\partial T}{\partial y} = Bx^2\theta' \tag{3.26}$$

$$\frac{\partial^2 T}{\partial y^2} = Bx^2\,\theta''\sqrt{\frac{c}{\nu}} \tag{3.27}$$

$$\frac{\partial T}{\partial x} = 2xB\sqrt{\frac{\nu}{c}}\theta \tag{3.28}$$

Substitute equations 3.26 -3.28 in equation (3.8):

$$2x^2B\sqrt{c\nu}f'\,\theta - x^2B\sqrt{c\nu}f\theta' = \frac{k}{\rho c}x^2B\sqrt{\frac{c}{\nu}}\theta'' - \frac{Q}{\rho\, c}x^2B\sqrt{\frac{\nu}{c}}\,\theta \tag{3.29}$$

Divide by $x^2B\sqrt{c\nu}$:

$$\frac{k}{\rho c\nu}\theta'' + f\theta' - 2f'\theta - \frac{Q}{\rho\, c^2}\theta = 0 \tag{3.30}$$

Where $Pr = \frac{\rho c \nu}{k}$ the Prandtl number and $\beta = \frac{Q\nu}{k\,c}$ is defined as heat generation /absorption parameter.

Equation (3.8) becomes:

$$\theta'' + Pr\, f\theta' - 2Pr\, f'\theta - \beta\, \theta = 0 \tag{3.31}$$

So, the non-dimensional equations are:

$$f'^2 - ff'' = f''' - \lambda f' - \frac{M}{(1+m^2)}\,(f' + mh) \tag{3.32}$$

$$f'h - fh' = h'' + \frac{M}{(1+m^2)}(mf' - h) \tag{3.33}$$

$$\theta'' + Prf\theta' - 2Prf'\theta - \beta\theta = 0 \tag{3.34}$$

3.2 Results and Discussion:

The various parameters that have been varied include the Hall parameter m, Magnetic field M, Ratio of kinematic viscosity to Darcy permeability constant λ, and heat source parameter β. These parameters are input into a computer program where each parameter is varied at a time.

Figure (3.1) shows that the increasing in Hall parameter m leads to an increase in the magnitude of primary flow velocity while figure (3.2) shows that the Hall parameter has opposite effect on secondary flow velocity. Figure (3.3) shows that the Hall parameter has no effect on temperature profiles.

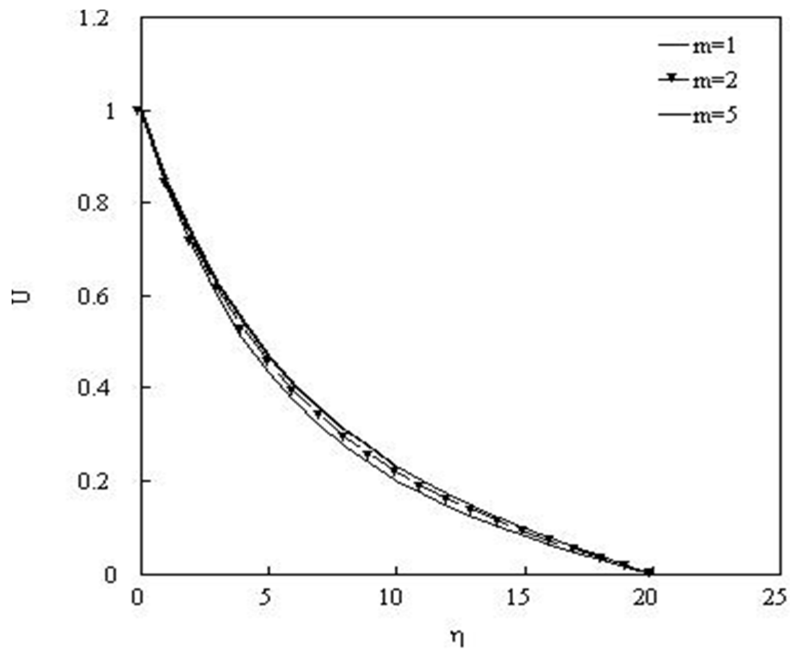

Figure (3.1) Effect of m on the primary flow velocity profiles U with M=1,β=5,λ=1

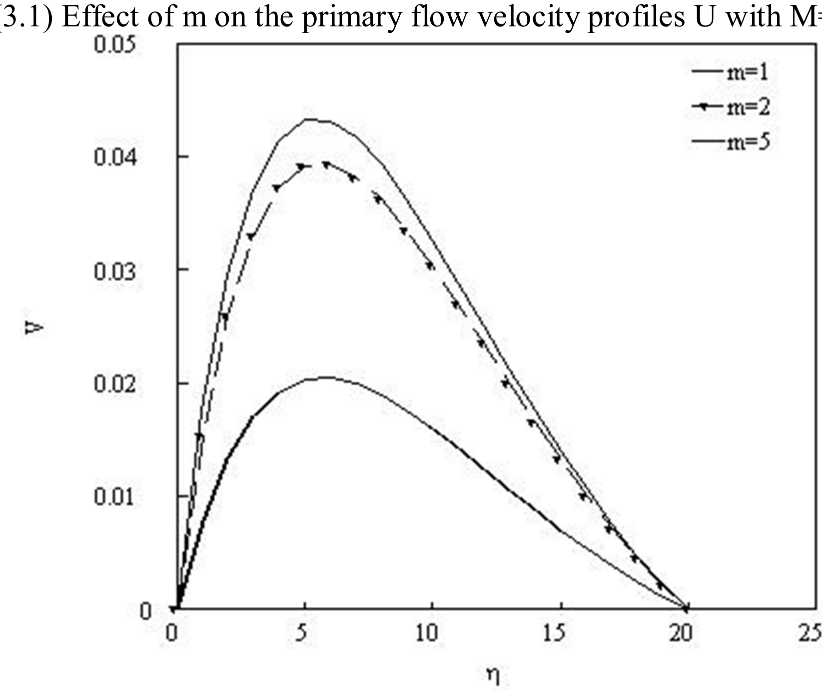

Figure (3. 2) Effect of m on the secondary flow velocity profiles V with M=1,β=5,λ=1

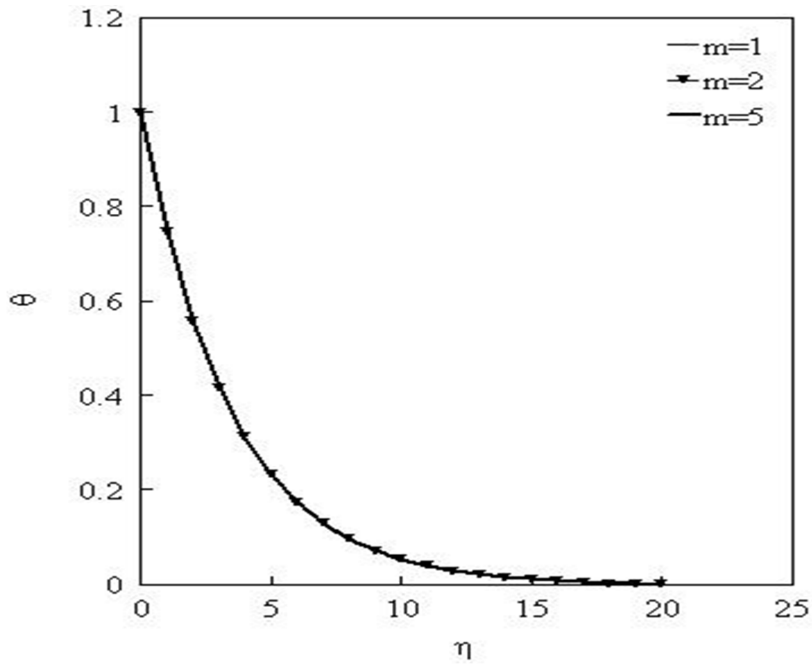

Figure (3.3) Effect of m on the temperature transfer profiles θ with M=1,β=5,λ=1

Figs.(3.4) ,(3.5) and(3.6) show the effects of the magnetic parameter M on the velocity and temperature profiles within the boundary layer. It is noticed that the increasing of magnetic field parameter M has a tendency to slow down the velocity of the fluid. Decreasing the velocity of the fluid slows down the movement of the species, while it increases the temperature profiles.

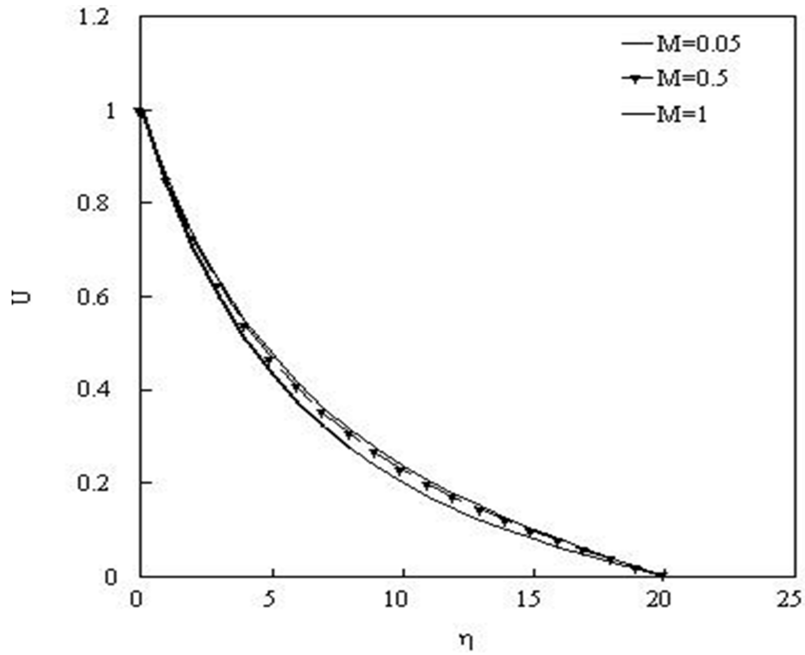

Figure (3.4) Effect of M on primary flow velocity profiles U with m=1, β=5,λ=1

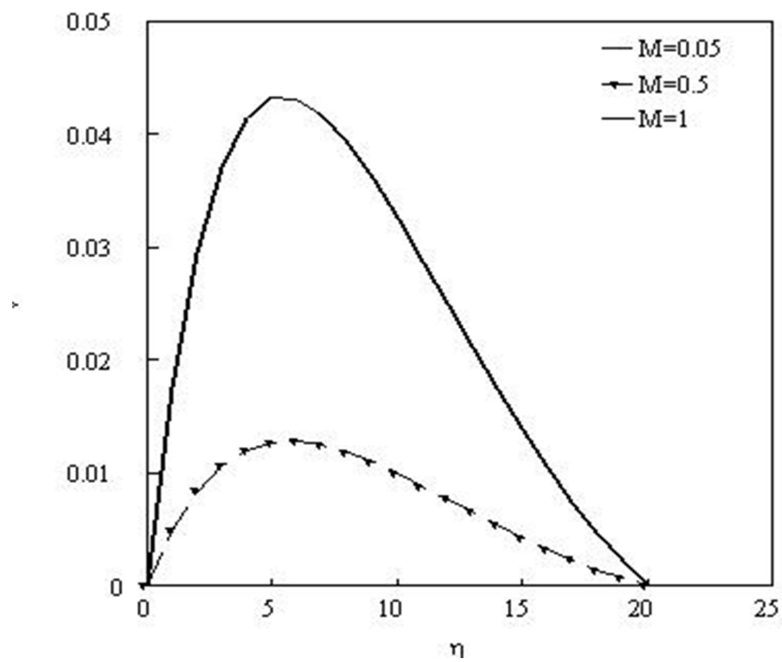

Figure (3.5) Effect of M on the secondary flow velocity profilesVwith m=1,β=5,λ=1

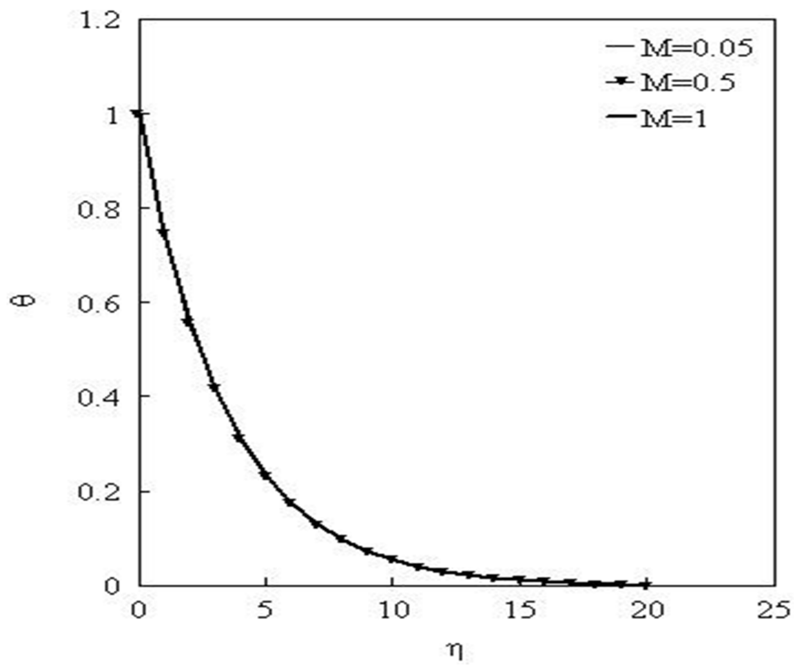

Figure (3.6) Effect of M on the temperature transfer profiles θ with m=1,β=5,λ=1

Figures (3.7) and (3.8) express that with the increase of the permeability parameter λ decreases the magnitude of primary and secondary flow velocities, is explained by the fact that increase in λ means a decrease in the size of the pores of the porous medium and this causes an increased resistance to the flow, leading to lower velocity. Figure (3.9) shows that increasing in λ tends to increase in temperature profile, this observation is due to the fact that increase in λ leads to thinner temperature boundary layer, thereby leading to an increase in the rate of heat transfer.

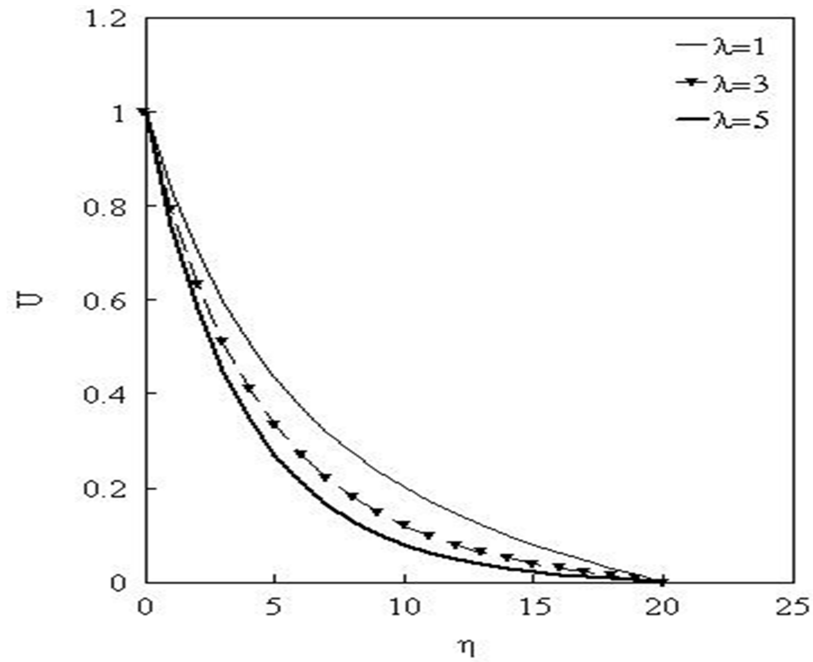

Figure (3.7) Effect of λ on primary flow velocity profiles U with m=1,β=5,M=1

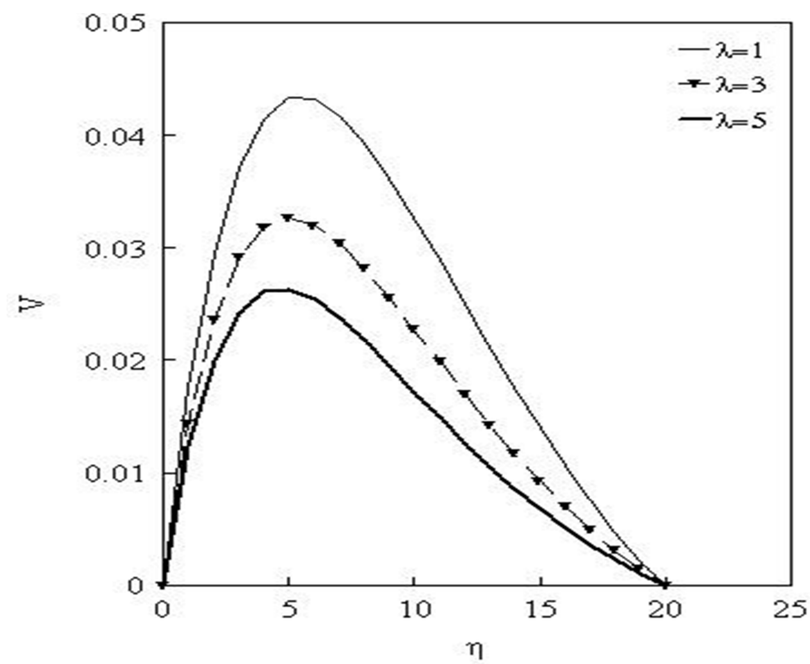

Figure (3.8) Effect of λ on the secondary flow velocity profilesVwith m=1,β=5,M=1

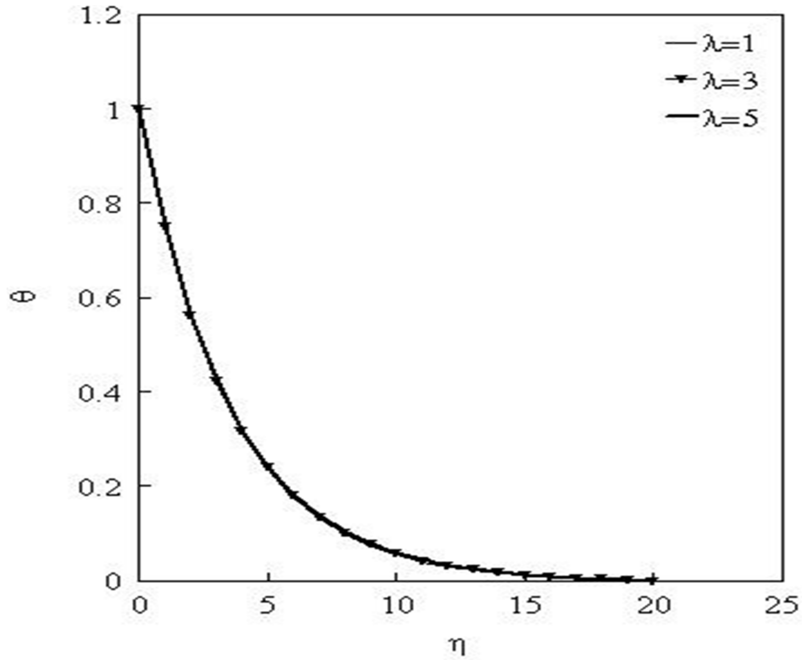

Figure (3.9) effect of λ on the temperature transfer profiles θ with m=1,β=5,M=1

The primary and secondary velocity doesn't change as the values of increase in figures (3.10) and (3.11). But the temperature in boundary layer decreases when magnitude of heat sourse rate increases in figure (3.12).

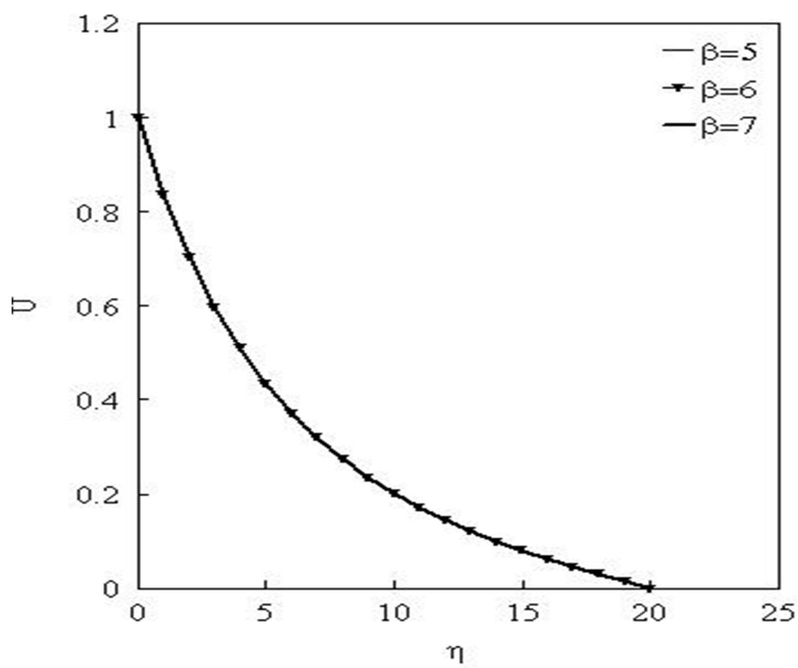

Figure (3.10) Effect of β on primary flow velocity profiles U with m=1,λ=1,M=1

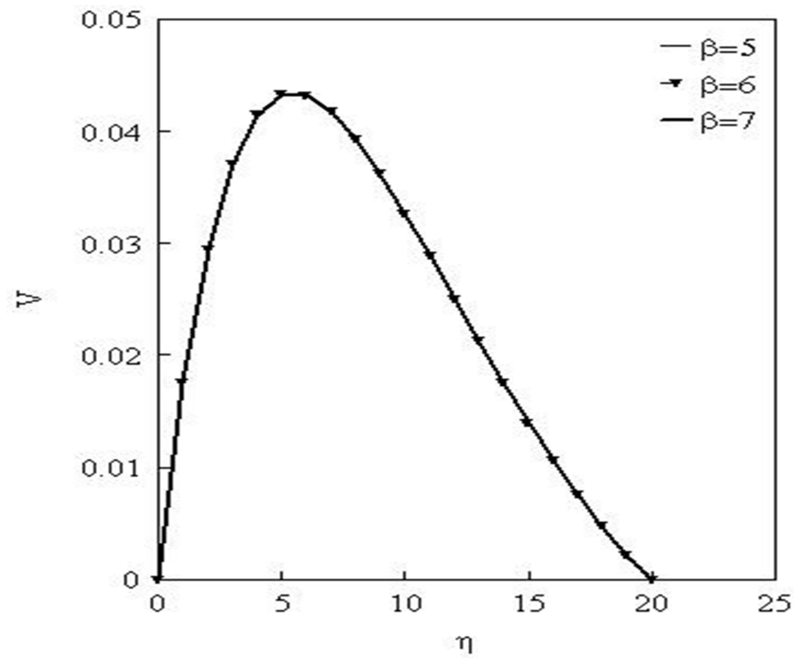

Figure (3.11) Effect of β on the secondary flow velocity profilesVwith m=1,λ=1,M=1

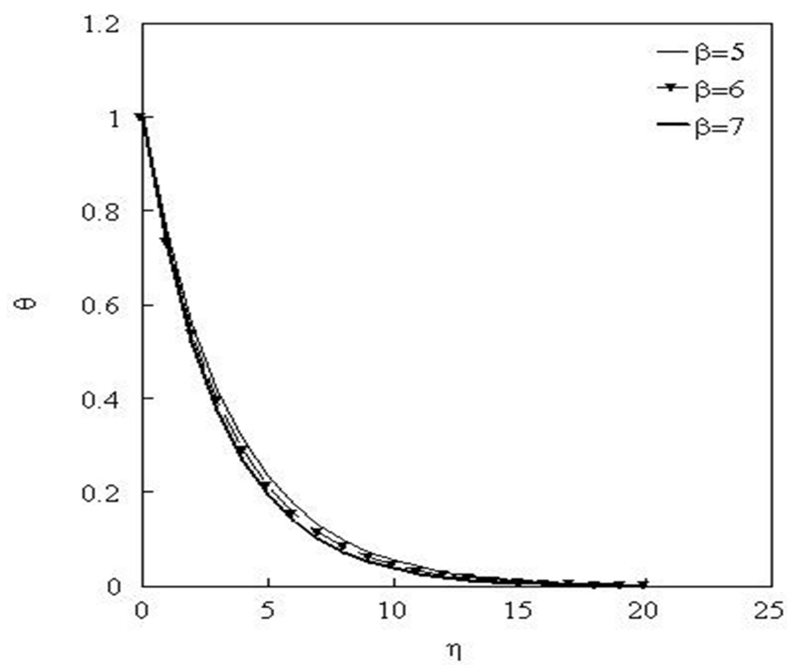

Figure (3.12) Effect of β on the temperature transfer profiles θ with m=1,λ=1,M=1

Table (3.1): Variation of dimensionless wall velocity gradients u and v and dimensionless rate of heat transfer θ with parameter of m , M , β and λ Pr=.72

m	M	β	λ	u'(0)	v'(0)	θ'(0)
1	0.05	5	1	-1.65035	0.000599388	-2.91966
1	0.5	5	1	-1.69552	0.0577776	-2.91777
1	1	5	1	-1.83227	0.207768	-2.91202
1	1	5	1	-1.83227	0.207768	-2.91202
2	1	5	1	-1.72858	0.180274	-2.91629
5	1	5	1	-1.66575	0.0910943	-2.91899
1	1	5	1	-1.83227	0.207768	-2.91202
1	1	6	1	-1.83227	0.207768	-3.10484
1	1	7	1	-1.88227	0.207768	-3.28443
1	1	1	1	-1.83227	0.207768	-2.91202
1	1	1	3	-2.38831	0.173973	-2.8909
1	1	1	5	-2.82693	0.152945	-2.87603

3.3 Conclusion:

In the present investigation, we dealt with the effect of Hall current on MHD flow of fluid over a horizontal porous sheet. The fluid is assumed to be electrically conducting .The highly non-linear coupled system of partial differential equations characterizing the flow and heat has been reduced to a coupled system of non-linear ordinary differential equations by applying a suitable transformation. The resulting system solved numerically. The obtained numerical results have been presented through the figures and tabular form to illustrate the details of the flow behavior and heat transfer and their dependence on the physical parameters that are involved in the present investigation:

1. Increase in magnetic parameter M results in decrease in the magnitudes of velocity component u. The magnitude of shear stress is proportional to velocity and since velocity profiles decrease with increase in M, the shear stress is expected to decrease. Thermal boundary layer thickness decreases with increase in M resulting to the observed increase in the wall temperature.
2. The shear stress in the secondary flow has its absolute value at m=1and approaches zero as $m \to \infty$, also increasing in Hall parameter m tends to increase the primary velocity component and to decrease the secondary velocity component.
3. While the increasing in heat resource parameter β has no effect on shear stresses $u'(0)$ and $v'(0)$. On the other hand the table illustrates that the wall temperature decreased as the value of parameters increased.
4. Both component of the shear stress are deceased, while the wall temperature is increased by increasing the permeability parameter λ.

Chapter Four

The Effect of Hall Current on Magneto-Hydrodynamic Flow and Heat Transfer over an Exponentially Stretching Sheet Embedded in a Thermally Stratified Medium

In Chapter 3, we studied the effects of Hall current on laminar boundary layer magneto hydrodynamic flow of an incompressible, viscous and electrically conducting fluid over stretching sheet embedded in porous media.

The purpose of the present chapter is to extend the flow and heat transfer analysis in boundary layer over an exponentially stretching sheet embedded in stratified medium using suitable transformation in absence of heat source. Third order ordinary differential equation corresponding to the momentum equation and second order ordinary differential equation corresponding to heat equation are derived. Numerical solutions of these equations are obtained by Newton's method.

4.1 Governing Equations and Analysis:

In this chapter the equations governing the MHD flow of an electrically conducting fluid over horizontal sheet in porous medium. The equations of the conservation of momentum and the equation of energy are derived in chapter two. This is followed by non-dimensionalizing process of the equations governing the flow. Newton's method used to approximate the solution to the governing equations is then discussed. Finally the results are discussed at the end of the chapter.

Consider the incompressible viscous laminar flow caused by moving sheet, which is placed in stationary fluid in absence of heat source. The flow is assumed to be in the x-direction which is chosen along the sheet and the y-axis perpendicular to it. A transverse magnetic field of strength B_0 is applied parallel to y-axis. The magnetic Reynolds number is taken to be small enough so the induced magnetic field is negligible in comparison with the applied magnetic field, so that:

$B_x = B_z = 0$ and $B_y = B_0$

The viscous and joule heating are taken into account. The generalized Ohm's law [38] including Hall current is given in the form:

$$J = \frac{\sigma}{1+m^2} \left(E + V \times B - \frac{1}{en_e} J \times B \right) \qquad (4.1)$$

Where J is the electric current density, (J_x, J_y, J_z) are the components of electric current density J, the equation of conservation of electric charge $\nabla . J = 0$ yields $J_y = 0$, V is the velocity vector, E is the intensity vector of the electric field, B is the induced magnetic vector , $m = \frac{\sigma B_0}{en_e}$ is the Hall parameter , e is the charge of an electron, and n_e is the number density of electrons. Neglecting polarization effect, the electric field E is given as E = 0.

So

$$J = (J_x, 0, J_z)\, ,\, B = (0, B_0, 0)\, ,\, V = (u, v, w) \qquad (4.2)$$

$$J_x = \frac{\sigma B_0}{(1+m^2)} (m\, u + v) \qquad (4.3)$$

$$J_y = \frac{\sigma B_0}{(1+m^2)} (m\, v - u) \qquad (4.4)$$

From the above equations, and considering the usual boundary layer and Boussinesq approximations, the equations of momentum and energy in porous medium are given below.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho(1+m^2)}(u + mw) \tag{4.6}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu - w) \tag{4.7}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{4.8}$$

Where u, v and w are the components of the velocity in the x, y ,and w directions, respectively.

The appropriate boundary conditions for the problem are:

$$At\ \ y = 0 \quad u = U_0, v = -V(x),\ w = 0,\ T = T_w(x) \tag{4.9}$$

$$u = v = w = 0,\ T = T_\infty(x) \text{ as } y \longrightarrow \infty \tag{4.10}$$

The solution of equations (4.5),(4.6) , (4.7)and(4.8) , satisfying the boundary conditions (4.9) and (4.10) is :

$$u = U_0 e^{\frac{x}{L}} f'(\eta),\ V = -\sqrt{\frac{\nu U_0}{2L}}\, e^{\frac{x}{2L}}\{f(\eta) + \eta f'(\eta)\},\ \eta = \sqrt{\frac{U_0}{2\nu L}}\, e^{\frac{x}{2L}} y\ ,$$

$$w = U_0 e^{\frac{x}{L}} h(\eta),\ \frac{T - T_\infty}{T_w - T_0} = \theta(\eta) \tag{4.11}$$

$$\frac{\partial u}{\partial x} = \frac{U_0}{L} e^{\frac{x}{L}} f' + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}}\, e^{\frac{x}{2L}} y f'' \tag{4.12}$$

$$\frac{\partial u}{\partial y} = U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}}\, e^{\frac{x}{2L}} f'' \tag{4.13}$$

$$\frac{\partial^2 u}{\partial y^2} = U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} f''' \tag{4.14}$$

$$\frac{\partial w}{\partial x} = \frac{U_0}{L} e^{\frac{x}{L}} h + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y \ h' \tag{4.15}$$

$$\frac{\partial w}{\partial y} = U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} h' \tag{4.16}$$

$$\frac{\partial^2 w}{\partial y^2} = U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} h'' \tag{4.17}$$

Substitute equations 4.12 – 4.14 in equation (4.6) ,we get:

$$U_0 e^{\frac{x}{L}} f' \left(\frac{U_0}{L} e^{\frac{x}{L}} f' + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y f''\right) - \sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{ f(\eta) + \eta f'(\eta) \} \left(U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} f''\right)$$

$$= \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} f''' - \frac{\sigma {B_0}^2}{\rho(1+m^2)} \left(U_0 e^{\frac{x}{L}} f' + m U_0 e^{\frac{x}{L}} h\right)$$

$$U_0 e^{\frac{x}{L}} f' \left(\frac{U_0}{L} e^{\frac{x}{L}} f' + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y f''\right) - \sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \left\{ f + \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y f' \right\} \left(U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}}\right.$$

$$\left. f''\right) = \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} f''' - \frac{\sigma {B_0}^2}{\rho(1+m^2)} \left(U_0 e^{\frac{x}{L}} f' + m U_0 e^{\frac{x}{L}} h\right)$$

$$\Rightarrow \frac{{U_0}^2}{L} e^{\frac{2x}{L}} f'^2 + \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y f' f'' - \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y f' f'' - \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} f f'' =$$

$$\frac{{U_0}^2}{2L} e^{\frac{2x}{L}} f''' - \frac{\sigma {B_0}^2}{\rho(1+m^2)} U_0 e^{\frac{x}{L}} (f' + mh)$$

Divide by $\frac{{U_0}^2}{2L} e^{\frac{2x}{L}}$ we get :

$$2f'^2 - f f'' = f''' - \frac{2\sigma \beta_0^2 L}{\rho(1+m^2) U_0 e^{\frac{x}{L}}} (f' + mh) \tag{4.18}$$

Equation (4.6) becomes:

$$f''' + ff'' - 2f'^2 - \frac{M}{(1+m^2)e^{\frac{x}{L}}}(f' + mh) = 0 \tag{4.19}$$

Substitute equations 4.15 -4.17 in equation (4.7), we get:

$$U_0 e^{\frac{x}{L}} f'(\frac{U_0}{L} e^{\frac{x}{L}} h + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y\, h') - \sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{ f(\eta) + \eta f'(\eta) \} (U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} h') =$$

$$\frac{{U_0}^2}{2L} e^{\frac{2x}{L}} h'' + \frac{\sigma {B_0}^2}{\rho(1+m^2)} (m U_0 e^{\frac{x}{L}} f' - U_0 e^{\frac{x}{L}} h)$$

$$U_0 e^{\frac{x}{L}} f'(\frac{U_0}{L} e^{\frac{x}{L}} h + \frac{U_0}{2L} e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y\, h') - \sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{2L}} \{ f + \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y\, f' \} (U_0 e^{\frac{x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} h')$$

$$= \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} h'' + \frac{\sigma {B_0}^2}{\rho(1+m^2)} (m U_0 e^{\frac{x}{L}} f' - U_0 e^{\frac{x}{L}} h)$$

$$\Rightarrow \frac{{U_0}^2}{L} e^{\frac{2x}{L}} f' h + \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y\, f' h' - \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} \sqrt{\frac{U_0}{2\nu L}} e^{\frac{x}{2L}} y\, f' h' - \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} f h' -$$

$$\frac{{U_0}^2}{L} e^{\frac{2x}{L}} f h' = \frac{{U_0}^2}{2L} e^{\frac{2x}{L}} h'' + \frac{\sigma {B_0}^2 e^{\frac{x}{2L}}}{\rho(1+m^2)} m U_0 e^{\frac{x}{L}} f' - \frac{\sigma {B_0}^2}{\rho(1+m^2)} U_0 e^{\frac{x}{L}} h$$

Divide by $\frac{{U_0}^2}{2L} e^{\frac{2x}{L}}$ we get :

$$2f' h - f h' = h'' + \frac{2\sigma \beta_0^2 L}{\rho(1+m^2) U_0 e^{\frac{x}{L}}} (mf' - h) \tag{4.20}$$

Equation (4.7) becomes:

$$h'' + fh' - 2f' h + \frac{M}{(1+m^2)e^{\frac{x}{L}}} (mf' - h) \tag{4.21}$$

The sheet is of temperature T_w and is embedded in a thermally stratified medium of variable ambient temperature T_∞ where $T_w > T_\infty$. It is assumed that $T_w = T_0 + be^{\frac{2x}{L}}$ $T_\infty = T_0 + ce^{\frac{2x}{L}}$, where T_0 is the reference temperature, b,c >0 are constants.

$$T = (T_w - T_0)\theta + T_\infty = (T_w - T_0)\theta + T_0 + ce^{\frac{2x}{L}}$$

$$= be^{\frac{2x}{L}}\,\theta + T_0 + ce^{\frac{2x}{L}}$$

$$\frac{\partial T}{\partial x} = \frac{b}{2L}e^{\frac{x}{2L}}\,\theta + \frac{b}{2L}e^{\frac{x}{2L}}\;\theta'\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\,y + \frac{c}{2L}e^{\frac{x}{2L}} \tag{4.22}$$

$$\frac{\partial T}{\partial y} = be^{\frac{x}{2L}}\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\,\theta' \tag{4.23}$$

$$\frac{\partial^2 T}{\partial y^2} = be^{\frac{x}{2L}}\frac{U_0}{2\nu L}\;e^{\frac{x}{2L}}\theta'' \tag{4.24}$$

Now substitute (4.22),(4.23) and (4.24) in equation (4.8) we get:

$$U_0 e^{\frac{x}{L}} f'\left(\frac{b}{2L}e^{\frac{x}{2L}}\,\theta + \frac{b}{2L}e^{\frac{x}{2L}}\;\theta'\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\,y + \frac{c}{2L}e^{\frac{x}{2L}}\right)$$

$$-\sqrt{\frac{\nu U_0}{2L}}e^{\frac{x}{2L}}\{\,f(\eta) + \eta f'(\eta)\,\}(\,be^{\frac{x}{2L}}\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\theta') = \frac{k}{\rho c_p}\,(be^{\frac{x}{2L}}\frac{U_0}{2\nu L}\;e^{\frac{x}{2L}}\theta''\,)$$

$$U_0 e^{\frac{x}{L}} f'\left(\frac{b}{2L}e^{\frac{x}{2L}}\,\theta + \frac{b}{2L}e^{\frac{x}{2L}}\;\theta'\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\,y + \frac{c}{2L}e^{\frac{x}{2L}}\right)$$

$$-\sqrt{\frac{\nu U_0}{2L}}e^{\frac{x}{2L}}\{\,f + \sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\,y\;\;f'\,\}(\,be^{\frac{x}{2L}}\sqrt{\frac{U_0}{2\nu L}}e^{\frac{x}{2L}}\theta') = \frac{k}{\rho c_p}\,(be^{\frac{x}{2L}}\frac{U_0}{2\nu L}\;e^{\frac{x}{2L}}\theta''\,)$$

$$\Rightarrow \frac{b}{2L}\mathrm{U}_0 e^{\frac{x}{L}} e^{\frac{x}{2L}} f'\theta + \frac{c}{2L}\mathrm{U}_0 e^{\frac{x}{L}} e^{\frac{x}{2L}} f' + \frac{b}{2L}\mathrm{U}_0 e^{\frac{2x}{L}} \sqrt{\frac{\mathrm{U}_0}{2\nu L}}\, y\, f'\theta' - \frac{b}{2L}\mathrm{U}_0 e^{\frac{2x}{L}} \sqrt{\frac{\mathrm{U}_0}{2\nu L}}\, y\, f'\theta' -$$

$$\frac{b}{2L}\mathrm{U}_0 e^{\frac{x}{L}} e^{\frac{x}{2L}} f\theta' = \frac{b}{2L}\mathrm{U}_0 e^{\frac{x}{L}} e^{\frac{x}{2L}} \frac{k}{\rho c_p}\theta''$$

Divide by $\frac{b}{2L}\mathrm{U}_0 e^{\frac{x}{L}} e^{\frac{x}{2L}}$

$$\Rightarrow f'\theta + \frac{c}{b} f' - f\theta' = \frac{k}{\rho c_p}\theta'' \tag{4.25}$$

Where prime denotes differentiation with respect to η only and the dimensionless parameters appearing in equations (4.27) - (4.29) are respectively $\mathrm{M} = \frac{2\sigma B_0{}^2 \mathrm{L}}{\rho \mathrm{U}_0}$ the magnetic parameter, $\mathrm{St} = \frac{c}{b}$ is the stratification parameter and $\mathrm{Pr} = \frac{\rho c_p}{k}$ is the prandtl number.

Equation (4.25) becomes:

$$\theta'' + \mathrm{Pr}(f\theta' - f'\theta) - \mathrm{PrSt}f' = 0 \tag{4.26}$$

So, the non-dimensional equations are:

$$f''' + ff'' - \mathrm{M} f' - 2f'^2 - \frac{M}{(1+m^2)e^{\frac{x}{L}}}(f' + mh) = 0 \tag{4.27}$$

$$h'' + fh' - 2f'h + \frac{M}{(1+m^2)e^{\frac{x}{L}}}(mf' - h) = 0 \tag{4.28}$$

$$\theta'' + \mathrm{Pr}(f\theta' - f'\theta) - \mathrm{PrSt}f' = 0 \tag{4.29}$$

4.1 Results and Discussion:

In this section we will analyze the results which that we got it from the numerical computation in the previous section for various values of suction parameter (S),stratification parameter (St),magnetic parameter(M) and Hall parameter(m) Figure(4.1) – (4.12) illustrate the results.

Figures (4.1)-(4.3) depict the effects Hall parameter, an interesting result observed from these figures that the cross-flow velocity gradually increases with the increase of $m \leq 2$ and the velocity decreases for $m > 2$. The values of m beyond which the flow behavior changes are considerable depending upon the choice of the magnetic parameter M. While the effect of the Hall current parameter m has an increasing effect on the dimensionless temperature θ (η) shown in figure (4.3).

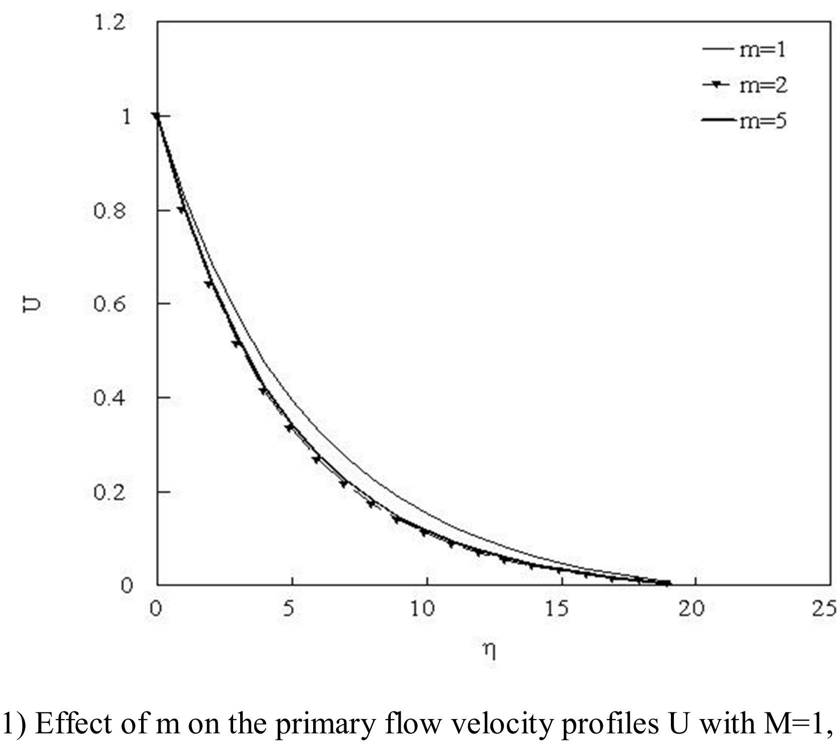

Figure (4.1) Effect of m on the primary flow velocity profiles U with M=1, S=1, St=0.1

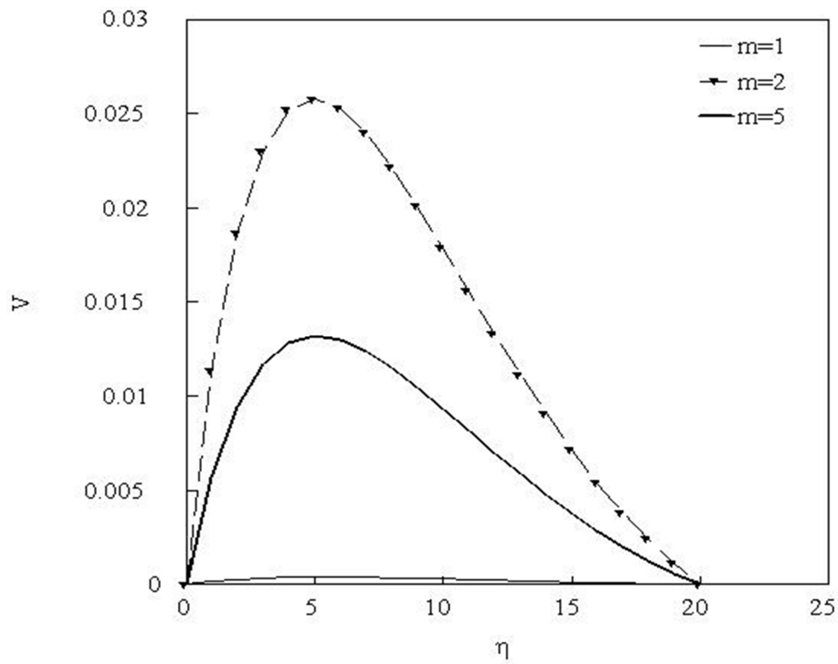

Figure (4.2). Effect of m on the secondary flow velocity profilesV with M=1, S=1, St=.1

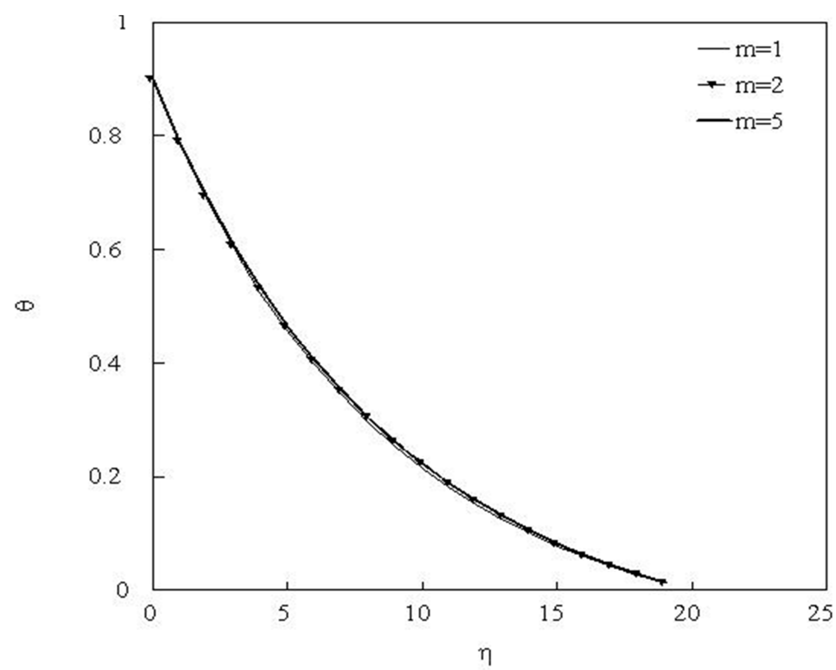

Figure (4.3) Effect of m on the temperature transfer profiles θ with M=1, S=1, St=0.1

Figure (4.4) represent the effect of variation of magnetic parameter on primary flow velocity. With increasing values of M, fluid velocity is found to decrease, because the

Lorentz force which opposes the motion of fluid increases with the increase in M. While figure (4.6) shows that increasing in M tends to increase in temperature.

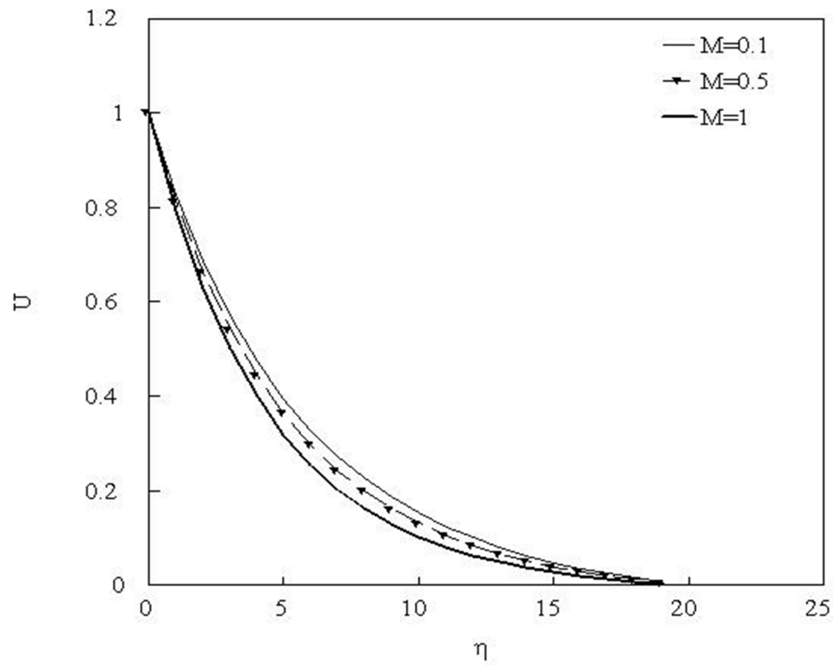

Figure (4.4) Effect of M on primary flow velocity profiles U with m=1, S=1, St=0.1

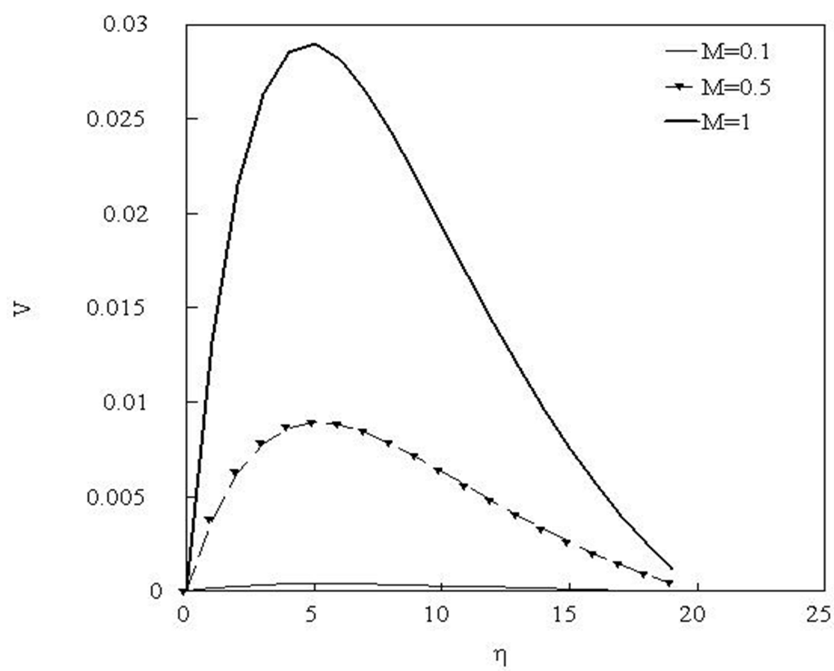

Figure (4.5) Effect of M on the secondary flow velocity profiles V with m=1, S=1, St=0.1

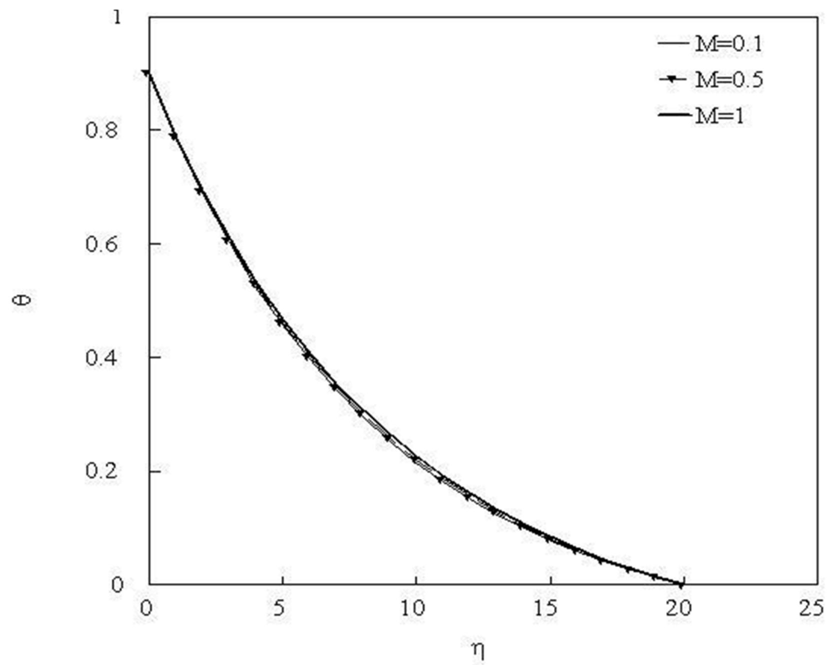

Figure (4.6) Effect of M on the temperature transfer profiles θ with m=1, S=1, St=0.1

Figures (4.7) – (4.9) depict the effects of suction parameter S on velocity and temperature, respectively, for exponentially stretching sheet. It observed that the velocity decreases with increasing suction parameter. The temperature suffers decrement as the suction parameter increases because the boundary layer decreases thickness. Further in absence of suction parameter the temperature profile becomes linear.

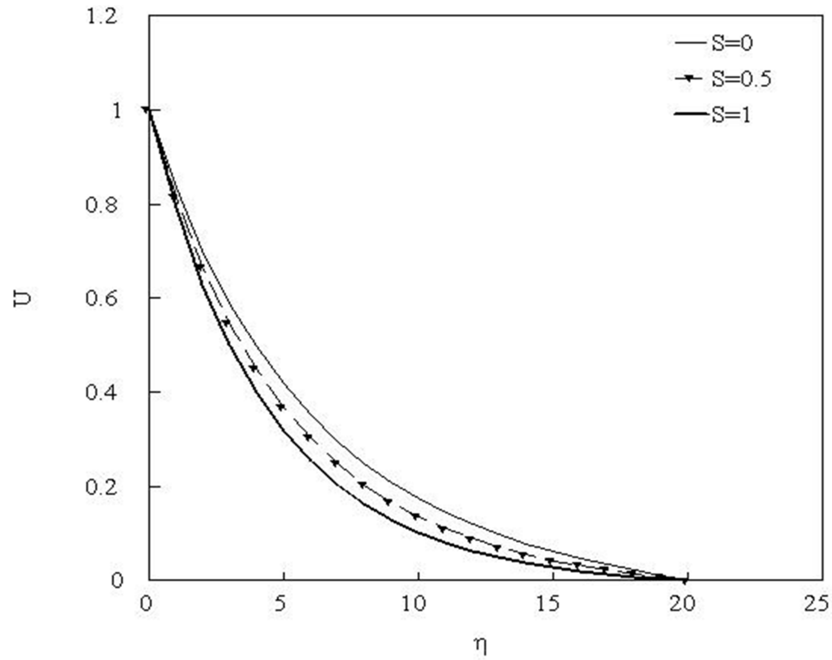

Figure (4.7) Effect of S on primary flow velocity profiles U with m=1,St=0.1,M=1

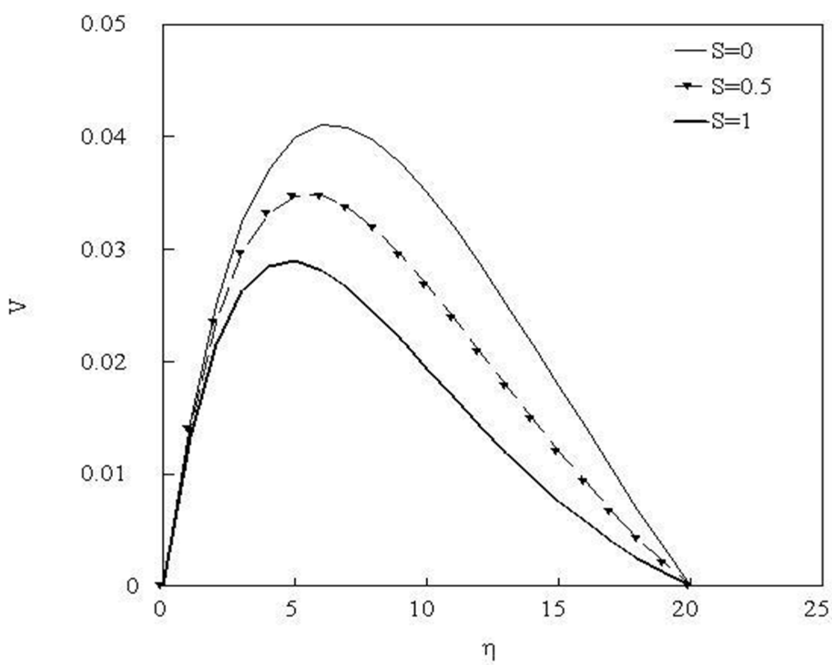

Figure (4.8) Effect of S on the secondary flow velocity profilesVwith m=1, St=0.1,M=1

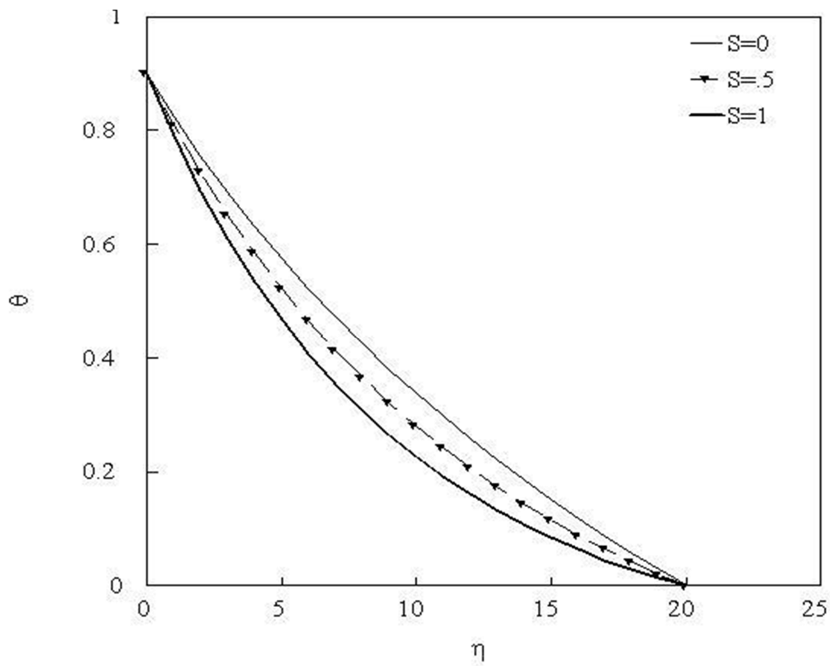

Figure (4.9) Effect of S on the temperature transfer profiles θ with m=1, St=0.1,M=1

Next we present the effect of thermal stratification parameter (St) on velocity and temperature in figures (4.10) – (4.12). It is clear that (St) does not effect on the velocity. But the temperature decreases as the stratification parameter increases. Since increase in St means increase in free steam temperature or decrease in surface temperature, thermal boundary layer thickness is therefore also decreased with an increase in St Values.

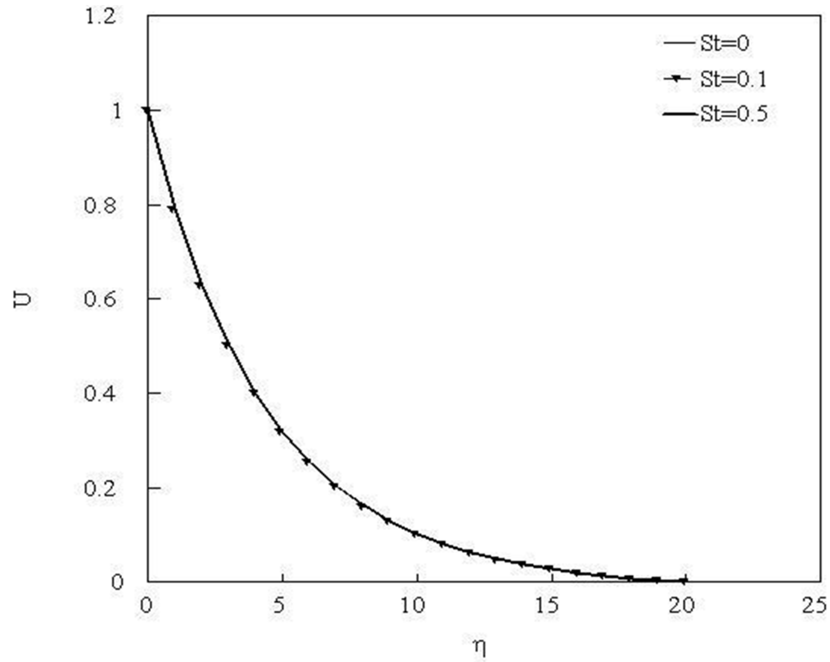

Figure (4.10) Effect of St on primary flow velocity profiles U with m=1, S=1, M=1

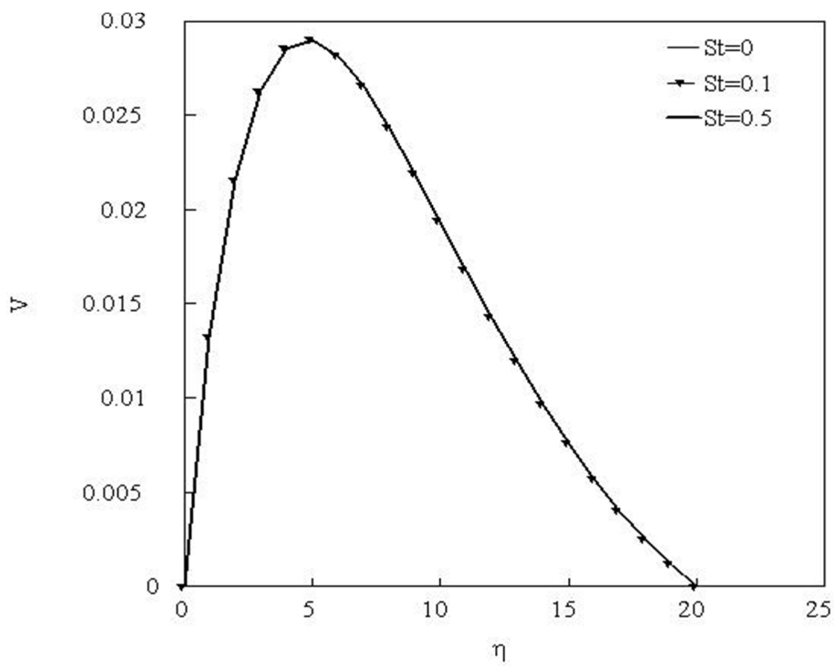

Figure (4.11) Effect of St on the secondary flow velocity profilesVwith m=1, S=1,M=1

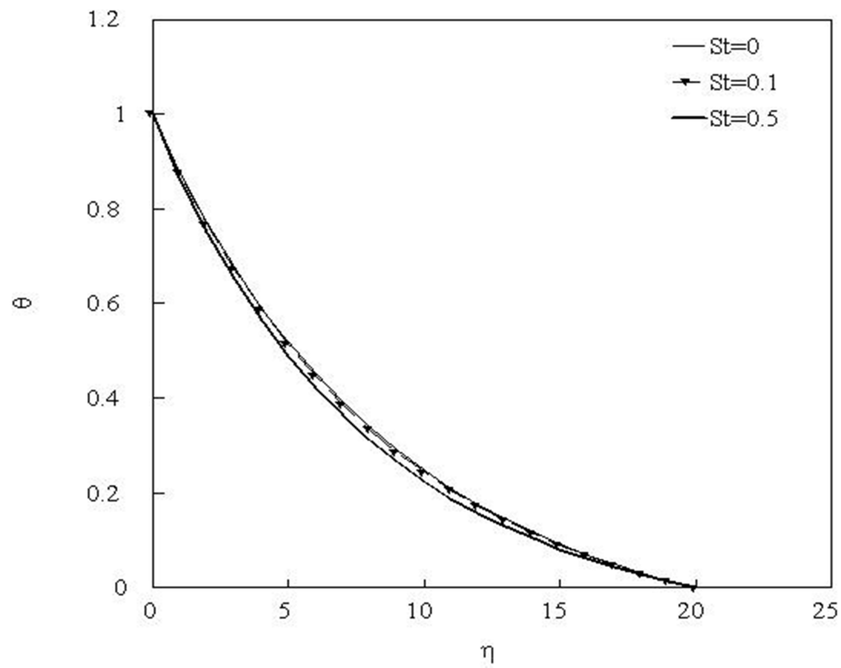

Figure (4.12) Effect of St on the temperature transfer profiles θ with m=1, S=1, M=1

Table (4.1) Variation of dimensionless wall velocity gradients u and v and dimensionless rate of heat transfer θ with parameters m , M , S and St for Pr=.72

m	M	S	St	u′(0)	v′(0)	θ ′(0)
1	1	1	0.1	-1.911448	0.00195801	-1.19861
2	1	1	0.1	-2.25816	0.137646	-1.17064
5	1	1	0.1	-2.20814	0.068721	-1.17473
1	0.1	1	0.1	-1.91448	0.00195801	-1.19861
1	0.5	1	0.1	-2.0837	.0457774	-1.1846
1	1	1	0.1	-2.34292	0.161716	-1.16417
1	1	0	0.1	-1.79116	0.168497	-0.789172
1	1	0.5	0.1	-2.04979	0.167161.	-0.962441
1	1	1	0.1	-2.34292	0.161716	-1.16417
1	1	1	0.0	-2.05303	0.175548	-1.28648
1	1	1	0.1	-2.05303	0.175548	-1.31527
1	1	1	0.5	-2.05303	0.175552	-1.42844

4.3 Conclusion:

The present study gives the numerical solutions for the effect of Hall current on steady MHD boundary layer flow and heat transfer over an exponential stretching surface embedded in thermally stratified medium in presence of suction.

The highly non-linear coupled system of partial differential equations characterizing the flow, heat transfer has been reduced to a coupled system of non-linear ordinary differential equations by applying a suitable similarity transformation. The resulting system is solved numerically by using the finite difference scheme along with the Newton's linearization technique. The obtained numerical results have been presented through the figures and in tabular form to illustrate the details of the flow behavior, heat transfer phenomena and their dependence on the physical parameters that are involved in the present investigation. From our computed numerical results we observed that:

1. Shear stress $u'(0)$ increased due the increasing in the values of Hall parameter m while the shear stress $v'(0)$ decreased..
2. Increasing values of magnetic parameter decrease the velocity components u and v and wall temperature.
3. The effect of suction as well as magnetic parameter on viscous incompressible fluid is to decrease the velocity.
4. The temperature decreases with increasing values of stratification parameter.

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التدفق الهيدرومغناطيسي لمائع في وسط مسامي

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الملخص

سوف ندرس في هذه الرسالة تأثير تيارهول على سريان هيدرومغناطيسي لسائل لزج غير قابل للانضغاط،حدودي رقائقي بوجود مجال مغناطيسيى ومصدر حراري داخل وسط مسامي.وسوف يتم تحويل معادلات الحركة وانتقال الحرارة إلى معادلات تفاضلية غير متجهة ليتم حلها بطرق التحليل العددي باستخدام برنامج الماثماتيكا. ثم مناقشة تأثير هذه العوامل على السرعة والحرارة وتمثيلها بيانيا عن طريق برنامج الهارفارد.